An Exploration of Number Sense, Motivation, and Mindset in Relation to Math Achievement for Middle School Students with Learning Disabilities

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Dedication

This dissertation is dedicated to my Father, my husband, and my family. Thank you for everything.
Acknowlegements

I feel so blessed to have three amazing committee members whom I am honored to acknowledge. I have received invaluable support and encouragement from my committee members throughout the journey of my dissertation. First, I would like to thank Dr. Tuckwiller for her guidance and encouragement. I could never fully express how much I am thankful for her gracious support as a mentor and an advisor. I learned so much from her dedication and compassion. I feel so grateful to have a role model who sincerely cares about her students. I would also like to honor my dissertation committee members, Dr. Dardick and Dr. Rice. Dr. Dardick helped me to explore and embrace the field of methodology. I would like to thank Dr. Dardick for his critical eye, which has challenged me and encouraged me to become an insightful researcher. Dr. Rice also inspired me to explore different perspectives deeply and guided me to write more clearly and think critically. Lastly, I would also like to express my gratitude to Dr. Taymans, who opened up a path to begin my journey, who continuously provided warm encouragement and genuine support.

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Abstract of Dissertation

An Exploration of Number Sense, Motivation, and Mindset in Relation to Math Achievement for Middle School Students with Learning Disabilities

Despite a U.S. emphasis on math achievement, low math performance of students with learning disabilities remains a concern. In an attempt to explore the factors related to student success in mathematics, previous studies have examined various factors related to math achievement (Akkaya, 2016; Dweck, 2006; Elliot & Dweck, 2005; Sasanguie et al., 2013; Yeager et al., 2016). Among these math achievement-related constructs are number sense, motivation, and mindset. While each of these factors has been found to have critical relationships to math achievement, there is a gap in this literature, as there is limited emphasis on middle school students with learning disabilities.

The purpose of this study was to explore mindset, motivation, and number sense in relation to math achievement for middle school students with learning disabilities. The present study used a mixed methodology with an integrated convergent design to explore these variables in depth. Each construct was assessed via a survey constructed of items selected from valid instruments. Interviews with students were analyzed in addition to the quantitative data. The results were compared and synthesized to enhance the understandings of the variables associated with math achievement for middle school students.

This study found that number sense was statistically significantly correlated to math achievement, and through the integrated analysis, that there were differences in motivation between the two groups (students with LD and students without LD), and that motivation was correlated to number sense and math achievement. This study also found
statistically significant differences between students with LD and students without LD in math achievement. Further, the mean difference of the scores on number sense between the two groups (students with LD and without LD) was significant.

In this study, students who reported that they had a growth mindset and were motivated to learn math tended to also discuss positive experiences in math. Furthermore, compared to students without LD, students with LD reported more negative motivation toward their experiences with and achievement in math. In addition, the IA numeracy scale (Gittens, 2015) was used to assess number sense for this study. The IA numeracy scale (Gittens, 2015) was strongly correlated with teacher evaluations of student number sense, indicating that the IA numeracy scale (Gittens, 2015) could be a reliable source for assessing number sense for future research, as well as researcher evaluations.
Table of Contents

Dedication ........................................................................................................................ iv

Acknowledgements ............................................................................................................ v

Abstract of Dissertation .................................................................................................. vi

List of Figures ................................................................................................................ xii

List of Tables ................................................................................................................. xiii

CHAPTER 1 ...................................................................................................................... 1

Overview ....................................................................................................................... 1

Statement of the Problem .............................................................................................. 8

Purpose and Research Questions .................................................................................. 9

Statement of Potential Significance ............................................................................ 10

Theoretical Foundation and Conceptual Framework .................................................. 11

Summary of the Methodology .................................................................................... 14

Limitations and Delimitations ..................................................................................... 16

Definition of Key Terms ............................................................................................. 17

CHAPTER 2 .................................................................................................................... 20

Introduction ................................................................................................................. 20

Overview of the Topic ................................................................................................. 20

Purposes of the Study ............................................................................................... 21

Methods of Literature Review .................................................................................... 21

Literature Review: Number Sense ........................................................................... 21

Literature Review: Motivation and Mindset .......................................................... 23

Description and Critique of the Scholarly Literature .............................................. 23
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Sense</td>
<td>24</td>
</tr>
<tr>
<td>Definition of Number Sense</td>
<td>24</td>
</tr>
<tr>
<td>Measurement of Number Sense</td>
<td>25</td>
</tr>
<tr>
<td>Synthesis of Reviewed Studies</td>
<td>34</td>
</tr>
<tr>
<td>Inferences about Number Sense for the Study</td>
<td>41</td>
</tr>
<tr>
<td>Motivation and Mindset in Math Achievement</td>
<td>46</td>
</tr>
<tr>
<td>Definition of Math-related Motivation</td>
<td>48</td>
</tr>
<tr>
<td>Measurement of Math-related Motivation</td>
<td>49</td>
</tr>
<tr>
<td>Synthesis of Reviewed Motivation Literature</td>
<td>51</td>
</tr>
<tr>
<td>Inferences about Math-related Motivation for the Study</td>
<td>55</td>
</tr>
<tr>
<td>Definition of Mindset</td>
<td>55</td>
</tr>
<tr>
<td>Measurement of Mindset</td>
<td>56</td>
</tr>
<tr>
<td>Synthesis of Reviewed Mindset Literature</td>
<td>56</td>
</tr>
<tr>
<td>Inferences about Mindset for the Study</td>
<td>59</td>
</tr>
<tr>
<td>Strengths and Limitations of Extant Literature</td>
<td>60</td>
</tr>
<tr>
<td>Theoretical or Conceptual Framework for the Study</td>
<td>61</td>
</tr>
<tr>
<td>CHAPTER 3</td>
<td>67</td>
</tr>
<tr>
<td>Introduction</td>
<td>67</td>
</tr>
<tr>
<td>Research Questions</td>
<td>67</td>
</tr>
<tr>
<td>Hypotheses</td>
<td>68</td>
</tr>
<tr>
<td>Overview of Methodology</td>
<td>69</td>
</tr>
<tr>
<td>Research Procedure</td>
<td>72</td>
</tr>
<tr>
<td>Researcher Reflexivity</td>
<td>86</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>--------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Revised Conceptual Framework</td>
<td>140</td>
</tr>
<tr>
<td>Limitations</td>
<td>142</td>
</tr>
<tr>
<td>Recommendations</td>
<td>143</td>
</tr>
<tr>
<td>Recommendations for Practice</td>
<td>143</td>
</tr>
<tr>
<td>Recommendations for Policy</td>
<td>146</td>
</tr>
<tr>
<td>Recommendation for Teacher Education</td>
<td>147</td>
</tr>
<tr>
<td>Recommendation for Future Research</td>
<td>147</td>
</tr>
<tr>
<td>Conclusion</td>
<td>148</td>
</tr>
<tr>
<td>References</td>
<td>150</td>
</tr>
<tr>
<td>Appendix: Measures</td>
<td>174</td>
</tr>
<tr>
<td>Tables</td>
<td>179</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1: Conceptual Framework for the Proposed Study .......................................................... 14

Figure 2: A Two-Way ANOVA of the effect of number sense and disability group and their interaction on math achievement ................................................................. 98

Figure 3: A Two-Way ANOVA of the effect of mindset and disability group and their interaction on math achievement ................................................................. 99

Figure 4: New Framework based on the Findings of this Study .............................................. 141
List of Tables

Table 1: *Number Sense Assessments for Different Age Groups* .......................................................... 179

Table 2: *Selected Characteristics of the Study Participants* .............................................................. 180

Table 3: *Definition of Number Sense* ................................................................................................. 181

Table 4: *Research Question and Design* ............................................................................................ 186

Table 5: *Analysis and Findings* ........................................................................................................... 193

Table 6: *Number of Students without LD and with LD for each Grade Level* ................................. 91

Table 7: *Group Statistics for each Variable (Number Sense, Motivation, Mindset, and Math* .......... 92

Table 8: *Correlation between each Variable (Number Sense, Motivation, Mindset, and Math Achievement) for both groups (students without LD and student with LD) combined* ................................................................. 93

Table 9: *Statistics of Teacher Ratings on Student’s Number Sense* ............................................... 94

Table 10: *Correlation Between Teacher Ratings of Three Teachers* .............................................. 95

Table 11: *Intraclass Correlation Coefficient* ...................................................................................... 95

Table 12: *ANOVA Analyses for Each Variable (Number Sense, Motivation, Mindset, and Math Achievement) for Groups (Students without LD and Students without LD)* ............................................................................................................................................................................ 96

Table 13: *Two-Way ANOVA with Groups and Number Sense with Math Achievement as Dependent Variable* .............................................................................................................................. 97

Table 14: *Two-Way ANOVA with Groups and Motivation with Math Achievement as Dependent Variable* .......................................................................................................................... 98

Table 15: *Two-Way ANOVA with Groups and Mindset with Math Achievement as Dependent Variable* ............................................................................................................................................................................ 99

Table 16: *ANCOVA with Number Sense as Covariate with Disability as Independent Variable and Math Achievement as Dependent Variable* ......................................................................................... 102

Table 17: *ANCOVA with Number Sense as Covariate with Mindset as Independent Variable and Math Achievement as Dependent Variable* ................................................................. 103
Table 18: ANCOVA with Number Sense as Covariate with Motivation as Independent Variable and Math Achievement as Dependent Variable .............................. 103

Table 19: ANCOVA with Disability as Covariate with Number Sense as Independent Variable and Math Achievement as Dependent Variable .............................. 104

Table 20: ANCOVA with Disability as Covariate with Motivation as Independent Variable and Math Achievement as Dependent Variable .............................. 104

Table 21: ANCOVA with Disability as Covariate with Mindset as Independent Variable and Math Achievement as Dependent Variable ...........................105

Table 22: Demographic and Disability Status for Students who Participated in the Interview .........................................................................................................107

Table 23: A Joint Display of Descriptive Statistics of the Variables and Main Themes of Motivation and Mindset from the Interview .............................................. 117

Table 24: A Joint Display with Different Levels of Number Sense and Their Perceptions of Motivation and Mindset related to Math Achievement ...........118

Table 25: A Joint Display with Different Levels of Math Achievement and Their Perceptions of Motivation and Mindset related to Math Achievement ..........122
CHAPTER 1: INTRODUCTION

Overview

Concerns with the low math performance of U.S. students have led to more rigorous standards for teaching and learning mathematics, as well as greater student accountability (NCES, 2015; NCTM, 2000). Math achievement has become more important for students to be prepared and gain competence, for a globally competing, advanced society. In addition, math difficulties have far-reaching negative consequences on children’s school careers (Fletcher & Vaughn, 2009). Poor mathematics achievement can have substantial consequences (Dyson, Jordan, & Glutting, 2013). Thus, math achievement is essential for overall life success and it needs to be prioritized. The National Council of Teachers of Mathematics (NCTM, 2000) developed standards for teaching math that reflect a greater emphasis on conceptual understanding and teaching for meaning. They also emphasized mathematical reasoning and problem-solving skills with real-world application (NCTM, 2000). These standards represent an acknowledgment of the importance of mathematical reasoning in real-world settings and the need for all students to achieve proficiency in mathematical achievement.

In an attempt to explore the factors related to student success in mathematics, studies have investigated the relationship between math achievement and a number of different factors, including anxiety (Ma, 1999), working memory (Swanson & Beebe-Frankenberger, 2004), and educational technology (Wenglinsky, 1998). The diversity of research foci relative to math achievement represents the complexity of factors that are related to math achievement. Among these math achievement-related constructs are number sense, motivation, and mindset. Each of these factors has been found to have
critical relationships to math achievement in various ways. A number of studies have suggested that number sense is the foundation of mathematics and is related to math achievement (Akkaya, 2016; Aunio et al., 2006; Cantlon et al., 2009; Ivrendi, 2011; Jordan et al., 2006; Locuniak & Jordan, 2008; Sasanguie et al., 2013; Schneider et al., 2008; Toll, 2016; Xu et al., 2005). Other research has suggested that students’ motivation for mathematics is a critical mechanism supporting academic achievement (Amrai, Motlagh, Zalani, & Parhon, 2011; Eccles & Wigfield, 2002; Elliot & Dweck, 2005; Gottfried, Cook, & Morris, 2005). Still others have found that a “growth mindset” can be a driving force in math achievement (Dweck, 2006; McCuchen, Jones, Carbonneau, & Mueller, 2016; O’Shea, Cleary, & Breen, 2010; Panunesku et al., 2015; Shen, Miele, & Vasilyeva, 2016; Yeager et al., 2016; Yeager & Walton, 2011).

These significant findings are an important body of knowledge informing current understandings of math achievement, and incorporate diverse perspectives and research foci. However, there is a gap in this literature, as there is limited emphasis for middle school students with learning disabilities. It is essential to understand the correlates of math achievement for students with learning disabilities, as they continue to have lower graduation rates and math standardized test scores when compared to their peers without disabilities (Gebhardta, Zehnera, & Hessels, 2014; Mazzocco, Devlin, & McKenney, 2008; NCES, 2015). In addition, although math achievement is likely comprised of complex interactions among a number of variables, to date, there have been no published studies examining the relationships among the cognitive, motivational, and emotional components of number sense, motivation, and mindset, and how they contribute to math achievement. This gap is especially alarming in the field of special education where
students with disabilities often have impaired number sense, low math motivation and fixed mindsets (Baird, Scott, Dearing, & Hamill, 2009; Dweck, 2006; Landerl, Bevan, & Butterworth, 2003; Mazzocco, Devlin, & McKenney, 2008; Sideridis, 2003). To address this gap, it is imperative to study of these meaningful variables in depth, to support these students in need.

**Number Sense**

**Definition.** When the phrase “number sense” is broken down into its component words, “number” and “sense”, each has its own meaning. The Merriam-Webster dictionary (2017) defines them as, “Number: a sum of units; a distinction of word form to denote reference to one or more than one; a word, symbol, letter, or combination of symbols representing a number” (Number. (n.d.) In *Merriam-Webster online dictionary*. Retrieved November 25, 2017, from https://www.merriam-webster.com/dictionary/number). “Sense” is defined as: a meaning conveyed or intended; conscious awareness; a motivating awareness; a particular sensation or kind or quality of sensation; capacity for effective application of the powers of the mind as a basis for action or response” (Sense. (n.d.) In *Merriam-Webster online dictionary*. Retrieved November 25, 2017, from https://www.merriam-webster.com/dictionary/sense)

Each term involves complex concepts on its own, but when the two concepts are combined, the complexity of the phrase “number sense”, is amplified. The complex characteristics of this concept have been described in various ways. Akkaya (2016) defined number sense as an understanding of different relationships between numbers and operations and the flexible use of these relationships. Number sense has also been described as an intuitive understanding of numbers, their magnitude, relationships, and
how they are affected by operations (Toll, Kroesbergen, & Van Luit, 2016). It can also be interpreted as an understanding of whole numbers, number operations, and number relations (Jordan, Kaplan, Nabors Olah, & Locuniak, 2006). There are some overlapping concepts in defining number sense, but scholars have interpreted number sense in diverse ways. However, in general, number sense can be interpreted as, “critical thinking applied to a quantitative context” (Gittens, 2015, pg. 2). According to Gittens (2015), number sense refers to the ability to make judgments based on quantitative information in a variety of contexts. It reflects how students communicate, process, and interpret quantitative information.

**A foundation for math achievement.** Humans have the capacity to understand quantity in infancy (Giralt & Bloom, 2000) and can learn how to count at an early age. When they enter school, they formally learn to add and subtract, multiply and divide, and further develop complex number concepts. However, before this learning can occur, children must deeply understand the meaning of numbers (Cantlon et al., 2009). Number sense in the early years of life is considered to be a foundation for later mathematical learning (Ivrendi, 2011). Number sense can be conceptualized as the understanding of numbers and operations (Jordan et al., 2009). Early number competencies develop in progression (Clements & Sarama, 2007). The core knowledge of numbers and number relations lead to number operations and more complex numerical knowledge (National Research Council, 2009). Students with good number sense develop a quantitative intuition that helps them solve problems in a flexible manner (Sood & Jitendra, 2013). They understand that numbers are representative of objects, magnitudes, relationships, and other attributes and are aware that numbers can be operated on, compared, and used

Number sense is one such entry-level competency. Although disagreements remain as to the exact definition of number sense, in general it involves students’ implicit understandings of the absolute and relative magnitude of sets of objects and of symbols that represent the quantity of these sets (Geary, Bailey, & Hoard, 2009). Student’s early number sense can be related to their ability to immediately identify the numerical value associated with small quantities, count small sets of objects and to add and subtract small quantities to and from these sets, and a proficiency in approximating the magnitudes of small numbers of objects and simple numerical operations (Dehaene, 1997; National Mathematics Advisory Panel, 2008). This intuitive sense of quantity and magnitude may provide the foundation for early mathematics learning in school (Geary, 2007).

On the other hand, children with poor number sense have difficulties in discriminating between quantities and are at risk for later failure in mathematics (Berch, 2005). Deficiencies in number sense are a core marker for severe and persistent learning disabilities in mathematics (Mazzocco, Feigenson, & Halberda, 2011). Once children are behind in mathematics, they tend to stay behind (Shalev, Manor, & Gross-Tsur, 2005). Thus, foundational number sense is critical in later mathematics achievement.

**Numbers sense for students with learning disabilities.** Preliminary evidence suggests that children with learning disabilities have a deficit in the fundamental understanding of number and magnitude (Landerl, Bevan, & Butterworth, 2003). Students with learning disabilities have historically performed worse in mathematics than students without disabilities, often particularly struggling with “automaticity of basic facts, computation problems, and problem solving” (Bouck & Yadav, 2008, p. 25). In the
Special Education Elementary Longitudinal Study (SEELS; Schiller, Sandford, & Blackorby, 2008), children with disabilities between the ages of ten and seventeen (N=5,400) were observed over a period of six years. Results showed that 60% of students with learning disabilities (LD) in segregated settings and 32% of students with LD in integrated classes achieved the lowest performance level in mathematics, below the 20th percentile (Schiller et al., 2008). In secondary school, the performance gap between the students with and without disabilities continued to widen. In ninth grade, the delay ranged from 3 to 4.9 years on average for students with LD (Blackorby, Chorost, Garza, & Guzman, 2003).

**Motivation**

Motivation cognitively and emotionally inclines students to practice and acquire skills (Wang & Barrett, 2013). It can also be described as, “the process whereby goal-directed activity is instigated and sustained” (Schunk, Pintrich, & Meece, 2008, p. 4). Studies have found that motivation is a possible predictor of how a student can perform academically (Hodis, Meyer, McClurre, Weir, & Walkey, 2011; Morgan, Fuchs, Compton, Cordray, & Fuchs, 2008). Many psychologists have emphasized the role of motivation in education because of its effective relationship with new learning, abilities, strategies and behaviors (Amrai et al., 2011). Motivational beliefs can also be considered as psychological mechanisms that influence students’ motivation to exert effort on learning tasks (Wigfield & Cambria, 2010; Wigfield & Eccles, 2000). Thus, it is a significant construct in academic achievement.

Previous research on academic motivation in childhood and adolescence has revealed a significantly steep decline for math (Gottfried, Fleming, & Gottfried, 2001).
Researchers found that achievement motivation and attitudes towards math were both directly and indirectly related to math grades (Singh, Granville, & Dika, 2002). In addition, evidence supported that math achievement is a significant contributor to the developmental decline in intrinsic math motivation (Gottfried, Marcoulides, Oliver, & Guerin, 2007). The middle-school years are a critical time to identify students who are struggling academically and to intervene (Balfanz, 2009). Especially for students who often face academic barriers, motivation is an important factor (Urdan & Turner, 2005).

**Mindset**

In the current educational context, a great deal of emphasis is placed on academic performance and instruction. However, students’ beliefs can also have a profound effect on their academic achievement (Dweck, 1999). Students can have different beliefs about their own intellectual abilities. Some believe that abilities cannot be improved much with practice or effort, and some believe that they can improve their abilities over time (Dweck, 1999). According to Dweck (2006), students’ mindsets can be described in two ways: fixed mindset and growth mindset. A fixed mindset is a belief that basic abilities such as intelligence cannot be changed and are an inherent, stable feature that is not responsive to intervention. A growth mindset is a belief that the basic abilities can be developed through effort and repeated practice. Mindset can influence the emotions that students experience as they encounter challenges. Studies have shown that students’ own views on intelligence, fixed or growth, influence how they respond to academic challenges (Blackwell, Trzesniewski, & Dweck, 2007; Yeager & Dweck, 2012). In particular, students with learning disabilities often tend to have a more fixed mindset compared to their typical peers (Hartmann, 2013). There is also an evidence that
mindsets can predict math and science achievement (Blackwell et al., 2007). Thus, mindset is a cognitive, emotional, and motivational construct in math achievement, especially for students with learning disabilities. It can have substantial impact on the student’s performance.

**Statement of the Problem**

To meet the global goal of workforce readiness, competitiveness, and equity in access, the United States has been devoting effort to accelerate mathematics instruction (National Research Council, 2011). Almost everyone uses some basic math skills on their job, such as simple tasks like counting, addition/subtraction, and multiplication/division, and perhaps proportions. Over 75% of all jobs require proficiency in simple algebra and geometry, and over 86% of jobs require simple addition or subtraction (Handel, 2016). Employment of mathematical science occupations is projected to grow 27.9% from 2016 to 2022, which is much faster than the average for all occupations (U.S. Bureau of Labor Statistics, 2018).

U.S. student progress in mathematics is assessed throughout the K-12 experience, from elementary to high school (NECS, 2015). Before students enter high school and learn more complex concepts, they need strong basic knowledge of mathematics to understand in depth. Middle school is a critical transitional period between elementary to high school, in which there is an opportunity to address foundational mathematics understandings, prior to the more rigorous demands of high school. Among these students who may need additional mathematical foundations instructions are students with learning disabilities who often need additional instruction to support their math achievement (Gebhardt, Zehner, & Hessels, 2014). In addition to standard mathematics
instruction, there are other constructs that can be addressed through targeted intervention to enhance achievement. For example, recent research suggests that cognitive, emotional, and motivational variables can also be critical contributors to math achievement (Akkaya, 2016; Amrai et al., 2011; Aunio et al., 2006; Cantlon et al., 2009; Eccles & Wigfield, 2002; Elliot & Dweck, 2005; Gottfried, Cook, & Morris, 2005; Ivrendi, 2011; Jordan et al., 2006; Locuniak & Jordan, 2008; Sasanguie et al., 2013; Schneider et al., 2008; Toll, 2016; Xu et al., 2005). It is critical to explore and understand how these cognitive, emotional, and motivational constructs intervene in math achievement for students with learning disabilities.

**Purpose and Research Questions**

The purpose of this study was to explore cognitive, emotional, and motivational components of mindset, motivation, and number sense in relation to math achievement. More specifically, the focus was to study how these variables (mindset, motivation, and number sense) contribute to the variance in math achievement among students with LD. The second study purpose was to understand how these variables (number sense, motivation, and mindset) differ between students with LD and students without LD. The study addressed the following research questions:

1. Do scores on measures of each of the three variables (number sense, math motivation, and mindset) differ between middle school students with LD and students without LD?

2. Do number sense, math motivation, and mindset predict math achievement in middle school students?
a. After controlling for disability status, are there differences in math achievement based on number sense, mindset, and motivation?

b. After controlling for number sense, are there differences in math achievement based on mindset and motivation?

3. Do teacher evaluations of student’s number sense correlate to scores on the IA numeracy scale (Gittens, 2015)?

4. How do middle school students with and without disabilities perceive mindset and motivation to relate to their experiences with and achievement in math?

5. How do middle school students perceive positive and negative experiences with achievement in math?

**Statement of Potential Significance**

The primary goals of this study were to explore cognitive, emotional, and motivational variables among middle school students with learning disabilities (LD) and understand how they contribute to math achievement. A review of the literature suggests a need to study the correlates of math achievement for middle school students with LD. While several variables have been found to be significantly related, there are no studies in which these constructs have been examined as a group to understand how they contribute to differences in math achievement for middle school students with LD. That is, despite a strong U.S. emphasis on math achievement, there are very few studies focusing on the foundational component of math achievement for middle school students: number sense. Prior studies of number sense have focused primarily on younger groups of students (Aunio et al., 2006; Irvendi, 2011; Jordan et al., 2006; Locuniak & Jordan, 2008, Sasanguie et al., 2013; Schneider et al., 2008; Toll et al., 2016; Xu et al., 2005). In
addition, while many studies indicate the role of motivation and mindset in math achievement for students in general, specific understandings of the influence of both of these constructs for middle school students with LD is limited. The present study aimed to examine the role of number sense in mindset, motivation and math achievement for middle school students with LD, to understand how each contributes to variance in math achievement among students with LD, and to explore potential differences in these variables between middle school students with and without LD.

The study findings offer important insights for the field of special education, particularly relative to predictors of math achievement for students with LD. This study also provided the first examination of this group of variables relative to math achievement for middle school students with LD and may suggest targets for interventions to support math achievement.

**Theoretical Foundation and Conceptual Framework**

In this study, I will reflect on the complex nature of number sense and use a model by McIntosh, Reys, & Reys (1992) as the number sense framework:

Number sense refers to a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations. It reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity. (p. 3)
Specifically, number sense is a set of skills that reflects foundational understandings in math. It is an ideal conceptual framework for the study of mathematics achievement. In this framework, the “inclination to use this understanding” suggests noncognitive constructs of math achievement (McIntosh et al., 1992). Number sense is a cognitive construct, which can interact with emotional and motivational constructs. There is evidence that self-concept affects number sense, and this provides evidence that a psychological orientation can affect academic achievement and cognitive aptitude (Geronime, 2009). It is thus important to explore the noncognitive skills embedded in math achievement.

Although cognitive abilities of students are important predictors of achievement, affective variables have emerged as salient factors affecting success and persistence in math (Singh, Granville, & Dika, 2002). In addition to number sense, a review of literature suggests that emotional and motivational constructs, such as motivation and mindset, are critical components of math achievement (Blackwell et al., 2007; Elliot & Dweck, 2005). Given the relationship of these noncognitive constructs to math achievement, I believe that it is important to gain further understanding of the constructs for middle school students with LD. Middle school is a developmental period where students transition through complex cognitive and emotional stages (Balfanz, 2009). It is a developmental period during which the adolescent is moving toward higher levels of cognition. Some students enter this period with a gap of foundational knowledge. As the gap widens, students may face challenges in terms of cognitive demand. These challenges can impact adolescent identity, competence, and motivation (Wigfield & Wagner, 2005).
While there is an emphasis on math achievement as an important target in the field of education, it is critical to find a way to carefully scrutinize factors related to mathematics achievement. We need to explore and determine strategies to improve it, especially for middle school students with LD. After a thorough review of the literature, I could not find a study that explicitly examined constructs of number sense, mindset, and motivation together as they relate to math achievement for middle school students with LD. Therefore, this study aimed to close the research gap and investigate cognitive (number sense), emotional, and motivational (mindset and motivation) constructs that are related to math achievement for middle school students with LD and provide expanded understandings of factors that influence math achievement in middle schoolers.

Specifically, in the initial conceptual framework for the study, I anticipated that a lack of number sense may interact with motivation and/or mindset and have a significant effect on math achievement in students with LD. In the conceptual framework for this study (see Figure 1), the bi-directional arrows suggest that the variables are related to math achievement. Data from this study can inform further studies in further understanding these variables as well as suggest revisions to this initial conceptual framework.
Figure 1. The hypothesized relationships between mindset, number sense, and motivation which are thought to be different based on disability status to math achievement.

Summary of Methodology

Students from an urban middle school in the Midwest region of the U.S. were recruited to participate in the study. The school is a public charter school that serves students both with and without special education needs. The participants were between sixth through eighth grade. I used convenience sampling at this school to recruit participants with and without learning disabilities. Participants completed a series of short questionnaires to measure their mindset, motivation and numbers sense, using well-established measures, with acceptable psychometrics. In addition, a selected sample of the participants participated in an interview about their perceptions of their mindset and motivation.

Each construct was assessed via a survey constructed of items selected from psychometrically sound instruments. To measure number sense, the IA Numeracy Scale
(Gittens, 2015) was used. This numeracy scale was conceptualized and developed by Insight Assessment, LLC (IA) (Gittens, 2015). The IA scale was developed to assess skills of analysis, inference, interpretation, explanation, and evaluation via items that do not rely on mathematical computations. To ensure developmentally appropriate mathematical content, the developers mapped the elements of the assessment to the draft Common Core Standards for Mathematics. The IA numeracy scale (Gittens, 2015) items require respondents to apply their critical thinking to either determine the optimal solution strategy or draw a reasonable conclusion from information provided. It consists of 11 items. As numeracy was connected to applied critical thinking, the items are more focused on reasoning rather than computation. In addition to the IA numeracy scale (Gittens, 2015), teachers (two math teachers and one special education teacher) evaluated student’s number sense, with a scale of one through five (1=low, 2=moderately low, 3=average, 4=moderately high, 5=high). The evaluations were compared to the scores on the IA numeracy scale (Gittens, 2015).

Student motivation data was collected via surveys administered in school. The surveys were adopted from a previous study on student motivation and math achievement with 8th grade participants (Simzar, Domina, & Tran, 2016). Motivation questionnaires were based on two widely-used frameworks: achievement goal theory and expectancy-value theory. The student surveys have 9 items, measuring for Achievement Goal and Self-Efficacy. All items were assessed using a 5-point Likert scale. To measure mindset, the Implicit Theory of Intelligence Scale (Dweck, 1999) was used. The scale consists of eight questions, answered on a 6-point Likert type scale. A selected sample of participants was interviewed on the topic of motivation and mindset. The students’ math
achievement scores were collected from the district annual testing records as measured by the Indiana Statewide Testing for Educational Progress Plus (ISTEP+).

The study used a mixed method in a convergent design framework, where both quantitative and qualitative data was collected during a similar time frame (Fetters et al., 2013). This study included an integrated convergent analysis. The results were compared and synthesized to enhance understandings of the variables associated with math achievement for middle school students. The integration of the data occurred through joint displays to simultaneously display the quantitative and qualitative results (Gutterman, Fetters, & Creswell, 2015). A joint display connected findings back to the theoretical framework of the study (Gutterman et al., 2015).

To answer the research questions, three statistical analyses were used in an effort to answer the research questions: correlation, ANOVA, and ANCOVA. I investigated the relationship between teacher evaluations of student’s number sense and the scores on IA numeracy scale (Gittens, 2015) with an interrater reliability test. To examine whether each variable (number sense, math motivation, and mindset) were significantly related, I ran a correlational analysis. I also conducted an analysis of variance (ANOVA) to find the differences in the means in relation to math achievement in middle school students. In addition to these analyses, I ran an analysis of covariance (ANCOVA) to control for the effects of covariates on the dependent variable of math achievement.

**Limitations and Delimitations**

Some limitations were anticipated in this study. The small sample size of convenience from a single site means the results of the study have limited generalizability. The sample, although representative of the site, is not representative of all middle school
schools or students with LD in general. Indeed, due to this school’s particular culture (discussed in Chapter 3), findings may not be relevant to students who were educated in different types of settings. However, to the extent that the site of research and the group of students serve similar populations, the findings may have a broader relevance. Another limitation was the characteristics of the survey measures. The participants completed surveys on number sense, motivation and mindset. These measures rely on the participants’ self-report on the questionnaires, and the requisite cautions of self-report data apply to this study. Furthermore, a single measure of math achievement was collected for this study, and the accurate assessment of math achievement is dependent upon the quality of this one measure. Finally, student with learning disabilities are compared to those without in this study. The presence of LD was not diagnosed clinically as part of this study procedure; rather, the LD identification in the school record was assumed to be accurate. In light of these limitations, the study findings may be relevant to help describe and predict the relationships among number sense, mindset, motivation and math achievement in middle school students with and without LD.

Thus, the aim of this study was to explore the relationships among the selected variables and designed to predict math achievement based on the cognitive, emotional, and motivational constructs of mindset, motivation and numbers sense. The findings did not provide causal relationships. However, the findings may potentially inform potential interventions targets for students with learning disabilities.

**Definition of Key Terms**

The following definitions are useful to understand the terms that are used throughout this document.
**Common Core Standards for Mathematics** (CCSSO, 2010) - a set of high-quality academic standards, created to ensure that all students graduate from high school with the skills and knowledge necessary to succeed in college, career, and life, regardless of where they live.

**IDEA** (IDEA, 2004) – the federal disability law entitling children with disabilities (birth to 21 years old) to receive a free appropriate public education (FAPE) in the least restrictive environment

**Implicit Theory of Intelligence** (Dweck, 1999; Paunesku, Yeager, Romero, & Walton, 2015) - conceptualizations of intelligence that individuals have for themselves, ideas that individuals have about what intelligence is and how it is formed

**learning disability** (IDEA, 2004) - a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations

**mindset** (Dweck, 1999) - a self-perception that people hold about themselves

**motivation** (Schunk, Pintrich, & Meece, 2008, p. 4) - “the process whereby goal directed activity is instigated and sustained”

**non-symbolic representation** (Smedt et al., 2013) - an analog, non-symbolic, and approximate system (e.g., comparing the numerosity of two groups of dot patterns)

**number line** (Sasanguie et al., 2013) - a line on which numbers are marked at intervals, or a standard model that is used to represent numbers
**number sense** (Gittens, 2015, pg. 2) - “critical thinking applied to a quantitative context”

**one-to-one correspondence** (Kearns, 2010) - the ability to match an object to the corresponding number and recognize that numbers are symbols to represent a quantity

**quantitative reasoning** (NCTM, 2000) - an application of basic mathematics skills to the analysis and interpretation of real-world quantitative information, also used in conjunction with numeracy

**self-efficacy** (Bandura, 1997) - an individual's belief in his or her capacity to execute behaviors necessary to produce specific performance attainments

**seriation** (Van Luit & Van de Rijt, 2005) - arranging objects in order by size, location or position

**symbolic representation** (Smedt, Noel, Gilmore, & Ansari, 2013) - quantities are represented as Arabic digits or number words (e.g., Arabic numerals such as “7” or number words such as “SEVEN”)

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**CHAPTER 2: LITERATURE REVIEW**

**Introduction**
Overview of the Topic

As a middle school math teacher, I witness students with learning disabilities struggle in mathematics. Yet, the students are “pushed” at a fast pace, to master all of the standards for the year. For some students, as they struggle with the pace and continue to fall behind, the math achievement gap grows ever wider. The lack of foundational skills leaves a serious gap in achievement for students with learning disabilities (LD). This widened gap leaves the students exhausted with the overwhelming demand and the frustration of being lost. As this unfortunate cycle continues, the students start to avoid math problems, lose interest, and feel helpless. I recognized similar and often more intense patterns of this cycle in many of my middle school students with LD. Within this demanding academic environment of secondary math instruction, it is crucial to interrupt the cycle of failure, frustration, and learned helplessness and support these students effectively to improve math achievement.

When students are motivated to learn, they become more engaged in learning and more likely to persist through challenges (Eccles & Wigfield, 2002). However, the relationships among math achievement and cognitive, motivational and emotional components for middle school students with LD are not well understood. While some studies have investigated each variable separately, there is very little data relative to these variables and math achievement for students with LD. The present study can potentially inform educators and the field of education about significant variables related to math achievement.

Purposes of the Study
The factors impacting math achievement are complex. To understand the variables that are implicated in math achievement, it is important to synthesize knowledge to date from prior studies examining each construct separately. Therefore, the purpose of this chapter is to describe current knowledge relative to each variable. This review of the literature is organized by research constructs to provide a clear overview of what is understood about each construct in isolation relative to math achievement. Next, the review integrates information relative to the constructs in terms of learning disabilities and math achievement. Finally, the conceptual framework on which the forthcoming study is grounded is presented to illuminate the links between the constructs for students with LD and provide a rationale for this exploratory work.

**Methods of Literature Review**

Several methods were utilized to locate literature regarding number sense, motivation and mindset. In initial searches, the *and function was used to locate studies examining all three variables (number sense, motivation, and mindset) for middle students with disabilities. This search returned zero relevant results. Subsequent searches included each of three constructs (mindset, motivation and number sense) in middle school students with LD, separately to locate studies of each construct. For each construct, a combination of search terms within multiple databases was used to locate applicable literature. At the time of this search, no prior studies had explored the three constructs together for middle school students; thus, the results of the separate searches are presented below.

**Literature Review: Number Sense.**

**Search terms: Number sense.** Identification of relevant studies was
accomplished using the search terms: number sense, numbers, assess number sense, examine number sense, investigate number sense, math, numerical processing, numeracy, and learning disabilities. The search was delimited by adding assess* number sense. These terms were used to search for any studies regarding the assessment of number sense and its correlates. The search was not delimited to middle school only, as there was only one study relative to number sense for middle school students (Gittens, 2015).

Sources. A computer-assisted bibliographic search for studies using the above search terms was conducted in Psychological Abstracts (PsycINFO), Educational Resources Information Center (ERIC) database, and Dissertation Abstracts. A search was also conducted on the World Wide Web using the search engine Google Search and employing the keyword terms number sense or assessing number sense. Bibliographies listed on specific websites related to number sense were searched also. In addition, reference sections of articles identified through the above means were searched for further relevant studies.

Selection criteria: Number sense. To be included in this synthesis, the studies had to meet three criteria: (1) studies had to include number sense measures, (2) practice designs for assessing number sense had to be included, and (3) the screening method of number sense had to be described. These criteria were included to explore how people have studied number sense and various instruments to assess number sense. Studies involving any age groups were included. Sixteen studies were located that met the selection criteria. Of those, six studies were excluded because of the lack of specificity regarding both the screening procedures and outcomes related to number sense. The remaining eleven studies were published between 2005 and 2016. Selected characteristics
of the study participants are summarized in Table 1 and the method of assessing number sense is noted.

**Literature Review: Motivation and Mindset.**

**Search terms.** Motivation and mindset. Identification of relevant studies was accomplished using the search terms: motivation, mindset, middle school, math achievement, growth mindset and motivation. The search was delimited by adding learning disabilities. These search terms were included to find literature on motivation and mindset together, and in relation to math achievement for students with LD.

**Sources.** A computer-assisted bibliographic search for studies using the above search terms was conducted in Psychological Abstracts (PsycINFO), Educational Resources Information Center (ERIC) database, and Dissertation Abstracts. A search was also conducted on the World Wide Web using the search engine Google Search and employing the keyword terms motivation or motivation and math. Bibliographies listed on specific websites related to motivation and math achievement were searched also. In addition, reference sections of articles identified through the above means were searched for further relevant studies.

**Selection criteria.** To be included in this synthesis, the studies had to meet three criteria: (1) studies had to include motivation and mindset, (2) students with LD had to be included, and (3) the correlation method of motivation and math had to be described. Studies involving any age groups were included.

**Description and Critique of the Scholarly Literature**

This section provides an overview of each selected construct on its own since they have not been explored in the aggregate relative to math achievement in the literature. It
will begin with an overview of number sense. Within this context, this review will analyze how number sense has been studied and related to other relevant correlates. This section will also investigate how motivation and mindset may relate to math achievement for students with disabilities. Finally, a conceptual framework will be presented to elucidate these hypothesized relationships among this group of variables and the related research questions.

**Number Sense**

The factors impacting math achievement are complex. One well-established correlate of math achievement is number sense. In order to understand the relationship between these two variables, it is important to first understand what number sense is and how it is related to math achievement. Since number sense is such a complex topic, numerous studies have investigated number sense in a variety of ways (See Tables 1 - 5).

**Definition of number sense.** In prior studies, researchers have created their own definitions of number sense, and research designs have depended on their interpretation of number sense. For example, Akkaya (2016) defined number sense as, “an understanding of different relationships between numbers and operations and the flexible use of these relationships” (p. 133). Based on this definition, number sense was assessed with the following sub-components: knowledge of numbers, use of multiple representations of numbers, ability to grasp number magnitudes (Akkaya, 2016). Number sense was also described as the ability to quickly understand, approximate, and manipulate numerical quantities (Schneider et al., 2008). Hence, number sense was focused on the “mental number line” as the core neurocognitive system underlying number sense. Another way that number sense has been described is as an intuitive
understanding of numbers, their magnitude, relationships, and how they are affected by operations (Toll et al., 2016). In understanding number sense in more depth, number sense has also been described as non-symbolic and symbolic. Jordan et al. (2006) characterized number sense as a recognition that numbers represent quantities and have magnitudes and an understanding of whole numbers, number operations, and number relations. To measure number sense, they assessed counting skills and principle, number recognition, number knowledge, and number operations. Numerical skills could be applied in the context of social and cultural factors, which can influence one’s development of number sense (Aunio, 2006; Irvendi, 2011). According to these descriptions, these factors potentially influence symbolic and/or non-symbolic development of number sense.

Table 3 summarizes each study’s definition of number sense as well as the measurement and assessment tools to quantify number sense. The overall concept of number sense from the studies generalized on the ability to explore numerical relationships in symbolic and non-symbolic representations. Thus, broadly speaking, number sense is not just counting numbers, but rather recognizing them and deeply understanding the meaning of them and what they represent. In general, it can be defined as, “critical thinking applied to a quantitative context” (Gittens, 2015, pg. 3). According to Gittens (2015), number sense refers to the ability to make judgments based on quantitative information in a variety of contexts.

**Measurement of number sense.** As reviewed in Chapter 1, number sense has been defined in the scholarly literature a number of ways. This is evident in the variety of measurement tools that have been developed to measure number sense or some portion of
the construct. Due to its complexity, the construct of number sense has a number of diverse dimensions to be conceptualized and investigated. The multidimensional nature of the construct has resulted in a scholarly literature in which a number of different elements of number sense have been explored in a variety of ways. Different studies have evaluated number sense utilizing a number of different lenses, and different researchers have chosen different tools to measure number sense. For example, some studies are based on the theory of “mental number line”, which is regarded as the core neurocognitive system underlying number sense (Sasanguie et al., 2013; Schneider et al., 2008, Xu et al., 2005). It is believed that the “mental number line” underlies a variety of behavioral competencies including estimating, computing, and efficiently using notational systems to solve mathematical problems (Schneider et al., 2008).

Other studies have used a battery of tests, such as the Utrecht Early Numeracy Test, which taps several aspects of young children’s numerical and non-numerical knowledge of quantity (Van Luit, Van de Rijt, &Pennings, 1999). It includes eight separate scales for assessing: concepts of comparison, classification, one-to-one correspondence, seriation, the use of number words, structured counting, resultative counting, and general understanding of numbers (Aunio et al., 2006). Likewise, there are other studies that have attempted to explore different ways to measure number sense (Jordan et al., 2012; Siegler & Ramani, 2009; Sood & Jitendra, 2013). For instance, the studies used numerical tasks or a battery of tests as their measurements. The intricate complexity of the concept of number sense has resulted in a variety of different studies and different measures of number sense. This variation in conceptualization of the construct requires that the concept and measurement of number sense be examined.
A number of standardized assessments to measure number sense have been developed for children. Since number sense is a complex topic, examining the validity of the assessments used to measure it is critical. There are several tests that measure number sense; for example, the Early Numeracy Test (Van Luit & Van de Rijt, 2005), Number Sense Brief Screener (Jordan et al., 2008), and Test of Early Numeracy (Clark & Shinn, 2004). These tests differ in terms of the breadth and depth of skills in number sense that they measure. However, they all aim to measure elements of number sense and also to identify students who do not display number sense at the expected developmental level.

**Early Numeracy Test.** The Early Numeracy Test (ENT) is designed to assess young children’s number sense. It is a 40-item, individually administered assessment designed to measure the early mathematical competence of students in preschool through first grade (Van Luit & Van de Rijt, 2005). It covers eight subtopics of number sense (1) concepts of comparison; (2) classification; (3) correspondence; (4) seriation; (5) using counting words; (6) structured counting; (7) resultative counting; and (8) general knowledge of numbers (Van Luit & Van de Rijt, 2005; Van de Rijt et al., 1999). The participants complete various tasks for each topic. For example, for the classification topic, participants group objects based on similarities and differences. The seriation topic involves ranking objects with given directions. For the structured and resultative counting topics, participants count objects with or without pointing or moving them. The general knowledge of numbers topic assesses participants applying mathematical knowledge to solve real life problems in drawings. The ENT is untimed and takes approximately 30 minutes to administer. The analysis of the English version of the ENT presented an internal consistency with a coefficient alpha of 0.83 for each test component (Van de Rijt
et al., 2003). The original version was created in Dutch, which had a slightly higher coefficient alpha value of 0.91 to 0.93 (Van de Rijt et al., 1999).

**Number Sense Brief Screener.** The NSB is a 33-item, individually administered assessment of number sense intended for use with kindergarteners and first graders (Jordan et al., 2008). This assessment was based on a study that aimed to create an assessment that could identify mathematics difficulties in children in kindergarten and first grade (Jordan et al., 2006). The researchers focused on skills that could be validated by research and also aligned with elementary school math curricula. Similar to ENT, the Number Sense Brief Screener (NSB) consists of 6 subtopics: 1) counting knowledge and principles, 2) number recognition, 3) number comparisons, 4) nonverbal calculation, 5) story problems, and 6) number combinations (Jordan et al., 2008; Jordan, Glutting, & Ramineni, 2009). The NSB is untimed and takes approximately 15 minutes to administer. The participants complete tasks for each topic. They are asked to count objects and identify incorrect or correct counting. They also identify two- and three-digit numbers for the number recognition task and compare different numbers. For the nonverbal calculation task, participants solve computation problems using chips, and solve real life problems that require simple computation.

Previous research on NSB suggests relatively promising reliability and validity of the test. Test-retest reliability in first grade ranged from 0.80 to 0.84 (Jordan et al., 2009; Jordan, Glutting, Ramineni, & Watkins, 2010). In addition, the internal consistency analysis yielded an alpha of 0.84 (Jordan et al., 2008). The researchers also supported the validity of the test. In this study, kindergarten performance on the NSB was predictive of third grade performance on the Math Achievement subtest of the Woodcock-Johnson III
(r = 0.65) (Jordan et al., 2008). In addition, a significant relationship was found between kindergarten performance on the NSB and third grade performance on a high-stakes state test (Jordan et al., 2010).

**Test of Early Numeracy.** The Test of Early Numeracy (TEN) is a set of curriculum-based measures of number sense (Clark & Shinn, 2004). It consists of four components that each take one minute to complete: Oral Counting, Number Identification, Quantity Discrimination, and Missing Number. Each brief subtest involves various tasks. For Oral Counting, participants count as many numbers as they can in one minute. For Number Identification, participants are asked to read as many numbers as they can in one minute, from a list of numbers ranging from zero to ten. For Quantity Discrimination, participants compare numbers and choose the bigger one. For the Missing Number task, participants fill in the missing number in a sequence of numbers.

A number of studies have investigated the technical adequacy of TEN probes. There is an evidence that the TEN is both reliable and valid for the assessment of the early numeracy skills of first graders (Clark & Shinn, 2004). Thirteen-week test-retest reliability was reported to range from 0.79 to 0.85 across subtests (Clark & Shinn, 2004). In addition, the subtests correlated well with other measures of early mathematical skills such as the Number Knowledge Test (r = 0.70 to 0.80) and the Woodcock-Johnson III’s Applied Problems subtest (r = 0.64 to 0.71) (Clark & Shinn, 2004). There was also a preliminary evidence for predictive validity of the TEN kindergarten probes with the Number Knowledge Test (r = 0.50 to 0.69; Chard et al., 2005). A predictive validity of the kindergarten probes with the Stanford 10 Achievement Test was also found (r = 0.31 - 0.46; Martinez, Missall, Graney, Aricak, & Clarke, 2009).
Each of these three measures are representative of number sense, but none of them are designed for older students in upper elementary or middle school. To date, no assessment of number sense has been designed for older students. Considering the importance of number sense as a foundational aspect of mathematical achievement, this lack of measures is surprising.

**Defining and Measuring Number Sense in Middle School Students**

The vast majority of previous research on number sense utilizes either pre-K or early elementary samples. The later elementary and middle school student population is underrepresented in the numeracy literature. This critical gap is due to a lack of valid and developmentally appropriate instruments for upper-elementary and middle school students (Gittens, 2015). There is limited data on middle school students’ number sense. In an attempt to fill the gap in the literature regarding number sense in students in upper elementary and middle school, Gittens (2015) introduced a scale for assessing numeracy as an applied form of critical thinking among children and adolescents. In her study, numeracy skill was assessed by the IA Numeracy Scale (Gittens, 2015), and these skills were consistently, strongly, and positively related to students’ math achievement and quantitative, verbal and non-verbal cognitive abilities (Gittens, 2015).

Gittens broadly defined numeracy as “critical thinking applied to a quantitative context” (Gittens, 2015, pg. 2), and operationally defined it as the ability to critically reason about quantitative information. According to Gittens, numeracy involves the ability to solve numerical and spatial-reasoning problems, draw inferences from quantifiable information in a variety of contexts, and reason probabilistically. Thus, numeracy is a foundational skill needed to interpret and evaluate quantitative information.
It also involves a recognition and understanding of how quantitative information is gathered, manipulated by counting and measuring, and represented visually context (Wiest, Higgins, & Frost, 2007).

**The IA Numeracy Scale.** The new numeracy scale was conceptualized and developed by Insight Assessment, LLC (IA), referred to as the IA numeracy scale (Gittens, 2015). The fundamental priority in developing the IA scale was to include items for skills of analysis, inference, interpretation, explanation, and evaluation, with no items that rely on mathematical computations. The assessment conceptualizes number sense as critical thinking relative to quantitative information. The IA Scale includes two levels, which are based on the typical developmental span characteristic of children and adolescents within the targeted grade-level range (grades 3-5 and grades 6-8) (Gittens, 2015). The framework for face-valid item creation for quantitative reasoning was based on the work of Nathan and Koedinger (2000) and other research on children’s arithmetic problem solving and algebraic reasoning (Carpenter & Levi, 2000; Nathan & Koedinger, 2000). To ensure developmentally appropriate mathematical content, the developers mapped the elements of the assessment to the draft Common Core Standards for Mathematics. According to the Common Core Standards for Mathematics for grades 6 through 8, students should be able to represent and solve arithmetic and algebraic expressions including one-variable equations; analyze quantitative and proportional relationships; engage in real-world problems requiring measurement, estimation, and functions; and reason inferentially about statistical variability, distributions and population data (CCSI, 2014a). Successful performance in these standards requires numeracy (Gittens, 2015).
**IA Numeracy Scale items.** The IA numeracy scale items require respondents to apply their critical thinking to either determine the optimal solution strategy or draw a reasonable conclusion from information provided (Gittens, 2015). It consists of approximately 30 items (15 items for each level of upper elementary and middle school student populations). There are two structural factors that have been shown to influence item difficulty: 1) start-unknown verses result-unknown position; and 2) word equations versus story problems for the two performance levels of the scale. For each level, there are more story problems than word problems. As numeracy was connected to applied critical thinking, the items are more focused on reasoning rather than computation, and the numeracy scale items include more start-unknown items than result-unknown items. The upper elementary level includes eight multiple-choice items and four response options (labeled A-D) that include the keyed (correct) answer and three closely related but incorrect distractors. The middle school level includes 11 multiple-choice items and five response options (labeled A-E) that include the keyed answer and four closely related but incorrect distractors.

The IA numeracy scale was created to engage students’ reasoning in relation to the mathematical content and quantitative reasoning expectations endorsed by the Common Core Standards (Gittens, 2015). The items frequently engage students in more than one content domain. The IA numeracy item scale includes specific critical thinking skills relative to the specified mathematical content domain (Analysis, Interpretation, Evaluation, Deduction, and Induction). The definitions of each area are based on the work of Facione and colleagues detailed in the publication of the APA expert consensus definition of critical thinking (Facione, Facione, Gittens, & Winterhalter, 2014). Analysis
involves identifying assumptions, reasons and claims, gathering detailed information from context, and closely examining the collected ideas and information. Interpretation reflects determining the meaning in a given context for a specific purpose. Evaluation refers to assessing the credibility of claims and the strength or weakness of arguments. Deduction is the process of drawing valid inferences. Induction is drawing inference from given information and context (Facione et al., 2014).

**Construct validity of the IA Numeracy Scale.** The primary focus of Gitten’s study was investigate the validity of a numeracy scale designed for students at the upper elementary and middle school level, and to explore numeracy as a habit of mind (Steen, 2010). The results of this study indicated that the numeracy scores were strongly and positively correlated with the ITBS math total score, with upper elementary participant scores correlated at .680, and middle school scores correlated at .574 with ITBS Math Total scores (Gittens, 2015). These correlational results suggest that 46% of the variance in ITBS overall math achievement for the upper elementary sample and 33% of the variance in the ITBS overall math achievement for the middle school sample was explainable in terms of numeracy as measured by this new instrument (Gittens, 2015). In addition, students’ numeracy scores on the IA numeracy scale were also strongly and positively associated with the CogAT composite score. These results indicated that 40 - 56% of the variance in the CogAT composite scores was explainable by scores on the numeracy scale (Gittens, 2015). However, the variables in CM3 were not statistically significantly associated, correlation ranging from .005 to .359. Gittens (2015) study provided empirical evidence to support that the IA numeracy scale is a valid assessment of numeracy skill for use with the targeted student populations (middle schoolers). In
addition, these results emphasized the importance of students’ numeracy to enhance mathematics achievement and cognitive abilities.

**Synthesis of Reviewed Studies**

The purpose of the number sense literature review was to synthesize what is known about the current conceptualization and measurement of number sense and its relationship to math achievement. However, the concept of number sense is multidimensional, and it is common to find different definitions of number sense across different researchers (Akkaya, 2016). The broadest definition of number sense includes a person’s sense of numbers and operations. Just as in the case of defining number sense, a common classification was not able to be established in defining its components. Past research has emphasized number sense as a complex process involving certain associations and skills related to numbers, and assessment of number sense has been diverse. It is important to note that all reported conclusions must be interpreted within the context of the study purpose, design, data collection approaches, and analytic processes employed in each study. Table 5 summarizes the purpose and the results of the reviewed studies.

**Mathematical associations.** Number sense performance is found to be one of the predictors of mathematical achievement (Sasanguie et al., 2013; Locuniak & Jordan, 2008; Toll et al., 2016; Akkaya, 2016; Jordan et al., 2006). The domain-specific numerical processes can be found as possible predictors for individual differences in mathematics achievement (Sasanguie et al., 2013). It was found that performance on both mathematics achievement tests was best predicted by how well children compared digits. There was also an association between performance on the symbolic number line
estimation task and math achievement scores for the general curriculum-based math test measuring a broader spectrum of skills. The results emphasize the importance of learning experiences with symbols for later math abilities. There was an evidence of number-specific and more general predictors of computational fluency in second grade relative to number sense (Locuniak & Jordan, 2008). Number sense tasks were varied which allowed to look at the relative importance of different but related quantitative skills to calculation fluency. The findings demonstrated the relation of early number skills to later, especially, number knowledge, and number combinations. Similarly, number sense can affect math performance in first grade and found that symbolic number sense was a predictor of mathematical performance in first grade (Toll et al., 2016). It was also found that a combination of visual working memory and number sense deficits (NSDs) can lead to the lowest performance on mathematics. These findings contribute to the knowledge on visual working memory and (non-)symbolic number sense as long-term predictors of mathematical performance in early primary school.

Considering the central role that number sense has in real life and significant relationship between number sense and mathematical achievement, a thorough investigation of the number sense performance of students and the determination of areas where support is necessary, such as multiple representations and magnitude of numbers, has become increasingly important (Akkaya, 2016; Jordan et al., 2006). Studies examined the subcomponents of number sense as potential clues to help design mathematical supports and/or interventions. The components of number sense in secondary school students were: number concept, multiple representations, understanding and using the equivalent expressions of numbers and figures, understanding the effect of operations on
numbers, and flexibility in making calculations (Akkaya, 2016). The participants completed the Number Sense Test, which is comprised of 50 multiple choice questions, in the 5 subcomponents of number sense. The means and standard deviations for the performance scores of the secondary school students were computed based on the number sense test and its sub-components. The results indicated that the component of multiple representations and magnitude of numbers were the most challenging for secondary school students. In addition, an eight-week number sense intervention for kindergartners was found to be effective (Jordan et al., 2006). The intervention consisted of targeted instruction addressing whole number concepts related to counting, comparing, and manipulating sets. They examined the effect of intervention on number sense, and used the Number Sense Brief (NSB) to assess counting skills and principles, number recognition, number knowledge, and number operations. They found significantly higher and meaningful adjusted outcome scores at posttest as well as at delayed posttest for the intervention group. The results were significant at posttest for the calculation problems subtest.

These studies are informative for math educators and clearly emphasize the role of number sense in mathematics. For example, number sense performance can help determine the areas in mathematics concept attainment and math achievement that would benefit from educational support. Studies provide evidence of the importance of learning experiences with symbols to later math abilities (Akkaya, 2016; Sasanguie et al., 2013). However, the mathematical correlations fail to address other factor that might affect mathematical achievement. They also do not describe the role of number sense in long-term mathematical development.
Nonmathematical associations. Number sense has been positively correlated to nonacademic variables, such as behavioral self-regulation in children (Aunio et al., 2006; Irvendi, 2011). Other studies have found other associations of number sense and reading, oral language, memory, and spatial skill (Locuniak & Jordan, 2008). For instance, there were significant relationships between nationality and age on Chinese and Finnish children’s number sense (Aunio et al., 2006). Children’s numerical and non-numerical knowledge of quantity was assessed with subtopics of: concepts of comparison, classification, one-to-one correspondence, the use of number words, structured counting, and general understanding of numbers. It was found that the natures of relational and counting skills are somewhat different; since relational skills reflect general numerical abilities, they are less influenced by direct teaching or language differences than counting skills. Unlike relational skills, counting skills rely on the use of a culturally based symbolic system. This suggests that the development of informal mathematical skills is universal, but that the pace of development/learning may vary.

Additional evidence exists that demonstrates the predictive power of behavioral self-regulation, family characteristics, and child characteristics on children’s number sense (Irvendi, 2011). To assess number sense, they administered the Assessing Number Sense Instrument (ANS) which consisted of items on number production, number identification, and counting. Using paper-pencil assessments with preschool and kindergarten students, they found relationships between specific number sense indicators such as age and behavioral self-regulation. They found that self-regulation, age, gender, mothers’ education level were significant predictor variables in the final regression model. This study suggests that association between learning and self-regulation of behavior
seems to be critical for the development of symbolic mathematical knowledge (symbolic knowledge develops through formal instruction). Similarly, there is an evidence that number sense is significantly related to skills on related tasks, such as reading, oral language, memory, and spatial skill (Locuniak & Jordan, 2008). To measure number sense, the number sense battery included varied number sense tasks on counting, number knowledge, nonverbal calculation, story problems, and number combinations. It was found that number sense measures contributed a significant amount of variance of the variables. Uniquely predictive sub-areas were active memory for numbers, number knowledge, and number combinations. The studies demonstrated number sense can be correlated to nonacademic variables that can be applicable to everyday life. They can also suggest starting point for further studies. However, the correlational results of these studies cannot demonstrate causal relationships among the variables.

**Developmental descriptions.** There is evidence that even infants have a measureable developmental understandings of number sense. For instance, six-month old infants have the capacity to represent numerosity in visual-spatial displays (Xu et al., 2005). Infants’ number discrimination ability has been assessed, and the results indicated that infants successfully discriminated the large-number displays but showed no evidence of discriminating the small-number display (Xu et al., 2005). They discriminated between arrays of 16 versus 32 discs, but not 16 versus 24 discs. Number sense has also been examined in terms of grade levels and gender (Akkaya, 2016). There is an evidence that number sense performance appears to increase in line with grade level, from 5th through 8th grade. However, there was no statistically significant difference in the number sense performance of secondary school students in terms of gender. These researchers designed
their studies in number sense with an emphasis of descriptive and developmental measures. The descriptive studies can provide a multifaceted approach for data collection. The statistical data can offer suggestions of meaningful relationship between variables.

**Physiological correlates.** There is a small body of evidence suggesting that number sense may map to physiological correlates. For instance, numerical tasks recruit different brain mechanisms (Cantlon et al., 2009). It was found that six and seven-year-old children and adults recruited different brain mechanisms to solve numerical comparisons. The fMRI scans provided evidence as subjects performed a number comparison task to measure the brain response. The results demonstrated that for adults, a region evoked significant response to both Arabic numerals and dot arrays, a significant numerical ratio effect included the pre-central gyrus and superior parietal cortex. Children exhibited ratio-dependent activity in the inferior frontal gyrus, the pre-central gyrus, and the thalamus. Unlike adults, children did not exhibit a significant ratio effect in superior parietal cortex. In addition, the eye movements can reflect children’s (1st through 3rd grade) use of the number line (Schneider et al., 2008). There was an evidence that the eye-tracking data reflected grade-related increase in estimation competence and children’s addition competence. The findings suggested that eye movements reflect children’s increasing knowledge about natural numbers, their interrelations, and ways of their spatial representation. These studies were designed to measure number sense through physiological attributes. The advantage of this design is that the evidence can be visible through differences in physical results. However, there are many experimental variables to control and the results may be difficult to replicate and interpret.
Closing comments on number sense measurement. Across all of the studies, researchers attempted to assess number sense in a way that represented their conceptualization of number sense (Akkaya, 2016; Aunio et al., 2006; Cantlon et al., 2009; Ivrendi, 2011; Jordan et al., 2006; Locuniak & Jordan, 2008; Sasanguie et al., 2013; Schneider et al., 2008; Tolls, 2016; Xu et al., 2005). These concepts and their assessment broadly included an examination of development in the ability to link between symbolic and non-symbolic representations of numbers. It also involved recognizing numerical values and fluently applying the knowledge in different representations. The sub-components of the assessments and also the number sense tasks were primarily measures of symbolic and non-symbolic representation of numbers.

The reviewed studies provide valuable insights about number sense. Through their own unique methods and designs, researchers assessed different conceptualizations and components of number sense. Since number sense is a complex concept to explain, some researchers have classified number sense in sub-components to measure it effectively. For instance, number sense was assessed in various ways, according to their own interpretation of number sense (Akkaya, 2016; Aunio et al., 2006; Ivrendi, 2011; Locuniak & Jordan, 2008). Each research team’s groups of sub-components broadly encompassed symbolic and non-symbolic representations of number sense. The majority of the studies focused on younger groups of participants, primarily preschool and elementary school students (Aunio et al., 2006; Ivrendi, 2011; Jordan et al., 2006; Locuniak & Jordan, 2008; Sasanguie et al., 2013; Schneider et al., 2008; Tolls et al., 2016;). In general, the studies have demonstrated that number sense involves multifaceted skills that involve multiple representations and magnitudes of numbers. Past
research has also emphasized the importance of number sense and its effect in one’s development, educationally, socially, and culturally. Through different interpretations, past studies have demonstrated that the development of numerical processing is progressive and can be supported. This suggests that number sense can be correlated with many other proscribed set of variables, and can be studied through making associations with other factors. However, there is a gap in the literature for the relationship between number sense with math achievement for middle school students with learning disabilities.

**Inferences about Number Sense for the Study**

The majority of previous research on number sense focuses on either pre-K or early elementary samples. The upper elementary and middle school student population is underrepresented in the numeracy literature. Research with younger children has suggested that number sense is clearly related to both academic and nonacademic development and achievement. However, there has been virtually no investigation of number sense and its correlates for older children. Considering the importance of number sense to math achievement, this gap is troubling especially for students with learning disabilities, a group who has significant challenges in both math achievement and number sense.

**Math achievement in the United States.** Government and professional groups have urged educators to help all students acquire mathematical preparedness for post-secondary education and employment (*Educate America Act*, U.S. Department of Education, 1994). The No Child Left Behind Act of 2001 (NCLB), outlined a national initiative for improving elementary and secondary education tied to high-stakes
assessments. In 2015, the successor of NCLB, the Every Student Succeeds Act (ESSA) continued the longstanding commitment to equal opportunity for all students. To ensure success for students, ESSA aims to advance equity by maintaining protection America’s disadvantaged and high-need students. ESSA also requires that all students are taught high academic standards, ready for college and career. In response to these pressures, the National Council of Teachers of Mathematics (NCTM, 2000) has called for curricular reform that emphasizes more problem-based learning. According to the NCTM, these problems should develop the skills and concepts of middle school students in (a) working flexibly with whole numbers, fractions, and decimals; (b) constructing and interpreting scale drawings; (c) converting units of measure; and (d) interpreting tables and graphs.

Most recently, the NCTM Principles to Actions: Ensuring Mathematical Success for All (2014) outlined essential Mathematics Teaching Practices, based on the belief that every student needs to develop mathematical understanding and self-confidence. The principles focus on reasoning and sense making, providing opportunities for all students. Specifically, one of the guiding principles focus on access and equity. According to NCTM, “Equitable access means high expectations, adequate time, consistent opportunities to learn, and strong support that enable students to be mathematically successful. Instead of one-size-fits-all practices and the differential expectations for students who are placed in different academic tracks, equitable access means accommodating differences to meet a common goal of high levels of learning by all students.” (NCTM, 2014, p. 4).

The results from the National Assessment of Educational Progress (NAEP) in 2015 indicated that eighth graders scored two percentage lower in 2015 than 2013, and
eighteen percentage higher compared to 1990, the first assessment year. However, the less positive findings showed that the average of NAEP math scores for students with disabilities was lower than those of students without disabilities (247 and 287 respectively). The most recent data show a downward trend in the scores of students with disabilities since 2011. In addition, 68% of students with disabilities received below “basic level” in 2015, compared to 65% in 2013 and 64% in 2011. Thus, the new standards call for a range of skills beyond procedural competency. A weak foundation of numerical knowledge and number sense can possibly lead to an urgent concern for students with LD who face more complex content in general education math classrooms.

**Gap in math achievement for students with learning disabilities.** Mathematical difficulties are widespread in school-age children and adolescents, and up to 20% of individuals have some form of mathematical learning disability (MLD) (Butterworth, Varma, & Laurillard, 2011; Dowker, 2009; Parsons & Bynner, 2005). Individuals with MLD have specific difficulties with numerical and arithmetic problem solving, despite age-appropriate schooling and absence of impairments in other cognitive domains (Butterworth et al., 2011).

Research suggests that there are significant gaps in math achievement between middle school students with learning disabilities and those without (Gebhardt et al., 2014; Mazzocco et al., 2008). Children with math learning disabilities (MLD) have demonstrated weaker rational number knowledge than children whose difficulty with rational numbers occurs in the absence of MLD (Mazzocco et al., 2008). In one study, participants were from a larger, longitudinal study of mathematical ability (Mazzocco & Myers, 2002). The sample was from 249 kindergarteners from previous study. The
participants were 6th, 7th, and 8th graders when they participated in the study in 2008. When the participants were in 3rd through 6th grades, the researchers used the Woodcock Johnson-Revised Calculations subtest (WJ-R-Calc) to classify the students into three groups: MLD, low average mathematics achievement (LA), or typical achievement of math (TA). In their study, the sample included 106 students in 6th – 8th grades: 12 with MLD, 18 in the LA group, and 76 in the TA group. All participants were tested individually during the spring with an assessment battery that included the Ranking Proportions Test (RPT) (Mazzocco et al., 2008). The measurement required the participants to compare and order decimals, fractions, or both. The results indicated that students with MLD were less accurate on the RPT than children in the LA and TA groups. There were significant interactions between participant group and subtests, $F(6, 101) = 5.22, p < .0001$ indicating that there was a statistically significant difference among the three groups (MLD, LA, and TA) on the RPT scores. During all three grades, fewer than half of the children with MLD correctly ranked all items on the easiest subtest, in contrast with 94.7% of the 8th graders in the TA group. These difficulties reflect a poor rational number sense, especially in students with MLD (Mazzocco et al., 2008). This suggests that students with MLD perhaps have basic conceptual deficiencies with fractions and decimals than students without MLD.

Students with learning disabilities also seem to have a gap in arithmetic skills (Gebhardt et al., 2014). The researchers investigated arithmetic skills in students with learning disabilities from fifth to ninth grade in Germany. They found that many students do not fully master basic arithmetic understanding in primary school. The researchers applied the newly developed Rasch scaled instrument as a reliable measurement of basic
arithmetical skills of students with Special Education Needs in the area of Learning (SEN-L) in secondary education in Germany. The categorization of students with SEN-L in Germany refers to significant academic difficulties in school, for which neither other disabilities nor lack of schooling can be found as cause (Lloyd et al., 2007). One third of the students with SEN-L, who have graduated from special schools, cannot handle numbers adequately and also have great trouble solving simple division tasks (Lehmann & Hoffmann, 2009). The lack of basic arithmetic skills is primarily responsible for mathematical difficulties in secondary school (Gebhardt et al., 2014). To investigate the students’ basic arithmetical skills, a group of 110 fifth to ninth graders participated in this study. They used several measures of basic arithmetical skills, for example, knowledge of quantity and operations rules. They found that there are many students who have not mastered arithmetical skills that are taught in primary school, which may affect the development of strong foundation. In their results, a significant interaction effect between development and grade was found in basic arithmetical skills: $F (1, 101) = 3.9, p = .01, \eta^2 = .14$. This indicates that students in lower grades improve their basic arithmetical skills over time while those in higher grades did not (and is some cases, a drop in performance). The results suggest that in higher grades, the basic arithmetic skills are no longer explicitly taught or consolidated. This may have an effect on students with LD in secondary school.

A computational fluency performance profile of 224 high school (9th to 12th grade) students with mathematics disabilities (MD) indicated that they were fluent only in computational skills at the second- and third-grade levels (Calhoun et al., 2007). This group of students was examined by grade-level expectancy (Grades 2—6) and skill area
(whole numbers: addition, subtraction, multiplication, division; rational numbers: fractions, decimals) using the Mathematics Operations Test—Revised (MOT-R). Lack of computational fluency was demonstrated on many items dealing with subtraction of multiple digits, items requiring regrouping, most multiplication and division items, and rational number items involving fractions and decimals (Calhoon et al., 2007).

Furthermore, students with mathematics LD tend to commit procedural errors, have difficulty organizing information, and experience both working memory and long-term memory deficits (Geary, 2004). These students also often have difficulty with basic computation and problem-solving exercises (Geary, 2004). These results suggest the importance of instructional practices that are effective for supporting secondary students with LD in their general education math curriculum. Given the difficulties experienced by many students with LD in math, it is imperative to support and close the gap in meaningful way.

For many middle students with LD, while they have demonstrated some understanding of the mathematics content, when they are required to answer multi-step or complex problem-solving questions, they struggle and feel overwhelmed. A synthesis of the research literature suggests that this might be the case because they have weak foundational skills in number sense. When required to understand questions and apply the concepts in various ways, lack of number sense may be a significant contributor to their difficulties.

**Motivation and Mindset in Math Achievement**

There are also noncognitive factors that influence academic performance. Motivation and mindset are emotional and motivational constructs that have been found
to be critical in academics (Blackwell et al., 2007; Dweck, 2006; Elliot & Dweck, 2005; Wigfield & Cambria, 2010). Social learning and expectancy theory research have demonstrated that students tend to actively approach activities when they feel confident and believe that they can succeed, whereas, some students avoid activities when they lack confidence of success (Kyriacou & Goulding, 2005). Previous research suggests that multiple psychological variables are related to learning, such as motivation and self-efficacy (Dweck, 1999; Eccles & Wigfield, 2002), and Dweck’s research in particular, supports the notion that students are more likely to persevere through academic challenges and succeed if they have a "growth" as opposed to a "fixed" mindset (Blackwell et al., 2007; Dweck, 1999; Farrington et al., 2012). A growth mindset is a belief that intelligence is developed through individual effort and grows with intentional practice (Paunesku et al., 2015). Academic mindsets can lead to academic perseverance, which can enhance academic behaviors that ultimately result in improved academic performance (Farrington et al., 2012).

Motivation is a related construct and has been implicated as a correlate of mindset (Blackwell et al., 2007). Motivation has been described as an individual’s beliefs in his or her ability to carry out a specific task, goals of the individual in doing the task, and the emotional response for carrying out the task (Amrai et al., 2011). The body of research investigating motivation and learning provides convincing evidence that students pay closer attention and invest more effort when they are more motivated (Krapp, 2002). Motivation and mindset are interrelated constructs, in which students’ perceptions can lead to self-evaluation and task-persistence (Chirila & Ticu, 2016). Although many studies have examined the effects of motivation and mindset on student achievement,
none have explicitly examined these variables concurrently for middle school students with LD. These emotional and motivational constructs are important for middle school students with LD, a group who need additional support for their academic progress.

**Motivation**

*Definition of math-related motivation.* One of the numerous ways to define motivation is, “the process whereby goal-directed activity is instigated and sustained” (Schunk et al., 2008, p. 4). In terms of math, motivation can be described as student’s process of persisting to solve the problem, to attain the goal (Deci & Ryan, 2000). Motivation is a possible predictor of how a student can perform academically (Hodis et al., 2011). One study provided evidence that students with low academic motivation have lower achievement and exert less energy toward completing their work (Urdan & Turner, 2007). Another study found that math achievement is a significant contributor to the developmental decline in intrinsic math motivation (Gottfried et al., 2007). Math motivation declined from ages 9 to 17 in math, as did math achievement (Gottfried et al., 2007). This suggests that students who experience recurrent academic failures in mathematics can feel increasingly less motivated to attempt their work. Repeated cycles of failure can then reinforce the idea that regardless of effort, the student will experience failure. Students with LD experience challenges and failure at a high rate (NECS, 2015). Thus, students with LD are more likely to have low motivation to participate in and persist through academic challenges (Morgan et al., 2008).

Motivational beliefs impact student’s effort on learning tasks (Wigfield & Eccles, 2000). Research supports that students develop intrinsic motivation when they are encouraged to develop their self-efficacy (Urdan & Turner, 2005). In terms of math,
when students are intrinsically motivated to learn math, they tend to be more persistent, and are confident in using more challenging strategies to solve mathematical problems (Lepper & Henderlong, 2000). Academic intrinsic motivation has been shown to foster positive dispositions toward mathematics. It also encourages students to develop self-efficacy and mathematical autonomy (Mueller, Yankelewitz, & Maher, 2011).

**Measurement of math-related motivation.** To measure motivation for math-related academic tasks, most researchers have employed surveys and questionnaires. There are several well-established measures, with acceptable levels of validity and reliability (reviewed below). On math motivation surveys, the items are worded to have students focus on mathematics lessons (e.g., I enjoy doing math.). One measure of math-related motivation is the Goal Orientation Questionnaire (Seegers, van Putten, & de Brabander; 2002). This questionnaire is comprised of three scales: 1) self-defeating ego-orientation, which assesses performance-avoidance goals; 2) self-enhancing ego orientation, which assesses performance-approach goals; and 3) task orientation, which measures mastery-approach goals. Students with mastery goals tend to focus on developing competence and are more likely to persist in learning activities (Harackiewicz, Barron, Tauer, Carter, & Elliot, 2000). Students with performance-approach goals focus on demonstrating competence (Harackiewicz et al., 2002; Pajares, Britner, & Valiante, 2000). In addition, students with performance-avoidance goals prioritize the avoidance of appearing incompetent.

Another measure of math motivation is the math scale of the Student Description Questionnaire (SDQ II; Marsh, 1990). SDQ II was designed to evaluate the multi-dimensional, hierarchical structure of self-concept for young adolescents in grades seven
through 12. The mathematics scale on SDQ II includes 10 items to measure “ability, enjoyment, and interest in mathematics and reasoning” (Marsh, 1990, p. 2).

In addition, a recent study measured student motivation for mathematics with 8th graders (Simzar et al., 2016). Motivation questionnaires were based on two widely-used frameworks: achievement goal theory and expectancy-value theory. Achievement goal theory explains why a learner engages in specific achievement-related behaviors (Kaplan, Middleton, Urdan, & Midgley, 2002), whereas expectancy-value theory predicts which activities individuals choose to engage (Wigfield & Eccles, 2000). The motivational measures were oriented toward achievement goals, self-efficacy, and subjective task value. The achievement goals were measured using three scales from the Patterns of Adaptive Learning Survey (PALS) (Midgley et al., 2000). All items were assessed using a 5-point Likert scale ranging from 1 (not at all true for me) to 5 (very true for me). The achievement goals included mastery goals, performance-approach goals, and performance-avoidance goals, similar to the construct measured on the Goal Orientation Questionnaire (Seegers et al., 2002). Mastery goals were assessed using items that focused on learning and understanding. Performance-approach goals included items that focused on demonstrating ability and outperforming others. Performance-avoidance goals were focused on not appearing incompetent. The standardized factor loadings and internal consistencies for these constructs ranged from acceptable to good (all standardized factor loadings were significant at \( p < .001 \). Chi-square = 3,883, df = 601, \( p < .001 \); CFI = 0.95; TLI = 0.95; RMSEA = 0.04; SRMR = .04.) (Midgley et al., 2000).

Thus, much of the math-related motivation assessment literature is characterized by measures that assess performance-approach, avoidance and mastery.
Synthesis of Reviewed Motivation Literature

Motivation is one of the critically important factors for academic learning and achievement across childhood through adolescence (Elliot & Dweck, 2005). Academic motivation is a student's desire to achieve in the academic subjects (Eccles & Wigfield, 2002). It is an individual’s beliefs in their ability to carry out a specific task, goals of the individual in doing the task, and the emotional response for carrying out the task (Amrai et al., 2011). One’s academic motivation can be reflected through his or her persistence and level of interest (Eccles & Wigfield, 2002). Motivation for academic achievement can be also interpreted as an inclination toward doing a task successfully and assessing the performance (Amrai et al., 2011). Many psychologists have emphasized the role of motivation in education because of its affective relationship with new learning, abilities, strategies and behaviors (Amrai et al., 2011). There is evidence that from childhood through adolescence, those with higher academic intrinsic motivation have been found to be more competent in school, generally evidencing significantly greater academic achievement, more positive perceptions of their academic competency, and lower academic anxiety (Gottfried et al., 2005).

Motivational beliefs can also be considered as psychological mechanisms that influence students’ motivation to exert effort on learning tasks (Wigfield & Cambria, 2010; Wigfield & Eccles, 2000). There are two widely used frameworks for studying motivational beliefs in academic contexts—achievement goal theory and expectancy-value theory. Achievement goal theory explains why a learner engages in specific achievement-related behaviors (Kaplan et al., 2002), whereas expectancy-value theory predicts which activities individuals choose to engage (Wigfield & Eccles, 2000).
Achievement goal theory includes mastery, performance-approach, and performance-avoidance goals (Dweck & Elliott, 1983). Students with mastery goals tend to focus on developing competence and are more likely to persist in learning activities, have greater interest in their classes, and seek help (Harackiewicz et al., 2000). Students with performance-approach goals focus on demonstrating competence, which is linked to positive associations with achievement and academic efficacy (Harackiewicz et al., 2002; Pajares et al., 2000). In addition, students with performance-avoidance goals prioritize the avoidance of appearing incompetent. These goals are typically associated with procrastination, withdrawal of effort, low performance outcomes, high anxiety, disorganized study habits, help-avoidance, self-handicapping, low achievement, and low interest (Midgley & Urdan, 2001; Urdan, 2004; Wolters, 2004). In the expectancy-value model of achievement, expectancy for success is defined as students’ beliefs about how well they will perform on an upcoming task, whereas task value is the degree to which a student believes that the academic task is worth pursuing (Wigfield & Eccles, 2000; Wigfield, Tonks, & Eccles, 2004). Task value is influenced by interest, attainment, utility, and cost (Simzar et al., 2007).

Task value has been found to predict mastery goals, and self-efficacy has been found to predict achievement goal (Liem, Lau, & Nie, 2008). A sample of 1,475 nine-year-old students participated in a research study that indicated that mastery and performance-approach goals were both positive predictors of deep learning and peer relationships. Mastery goals were also negatively associated with task disengagement and positively associated with learning. In contrast, performance-avoidance goals were a positive predictor of learning and task disengagement but a negative predictor of peer
relationship (Liem, Lau, & Nie, 2008).

Previous research on academic intrinsic motivation from childhood and adolescence has revealed a significant decline for the subject areas, in which, math was the area with the greatest and steepest decline (Gottfried et al., 2001). Researchers examined a sample of middle school students in the U.S. and found that achievement motivation and attitudes towards math were both directly and indirectly related to math grades (Singh, Granville, & Dika, 2002). Students’ attitudes towards math, achievement motives, and goal orientations also predicted math performance (Steinmayr & Spinath, 2009). In addition, math achievement is a significant contributor to the developmental decline in intrinsic math motivation (Gottfried, Marcoulides, Oliver, & Guerin, 2007). It was found that math motivation declined from ages 9 to 17 in math, as did math achievement (Gottfried et al., 2007). The study supported that math achievement significantly contributes to the decline in math motivation over the age range.

A significant association was found between algebra placement and student motivation for mathematics (Simzar et al., 2016). The students’ achievement goals, expectancy, and task value for students in eighth-grade algebra were compared to those of peers placed in lower-level mathematics courses. All students reported an overall decline in performance-approach goals over the course of eighth grade, but previously high-achieving students reported an increase in these goals (Simzar et al., 2016). The results indicated that while previously high-achieving students may benefit motivationally from eighth-grade algebra placement, placing previously average- and low-performing students in algebra can potentially undermine their motivation for mathematics.
Studies have shown that motivation for learning is positively influenced by self-regulated learning behavior and academic achievement (Peetsma & Van der Veen, 2013). In contrast, avoidance orientation is the opposite of approach orientation, which promotes students’ motivation and learning. Studies have found negative relationships between development of performance-avoidance orientation and school motivation (Yeo, Sorbello, Koy, & Smillie, 2008). They also found that students’ well-being in the school situation is negatively related to their avoidance orientation (Tuominen-Soini, Salmela-Aro, & Niemivirta, 2008). The motivators can be categorized into three parts: value, expectations and affective components (Peetsma, Hascher, van der Veen, & Roede, 2005). It involves goal orientations, beliefs about ability, and emotional reaction to learning. In the past, it has been found that students in the lowest level of secondary education show more avoidance orientation than students in the higher levels of secondary education (Peetsma, 1996). Groups of different avoidance-orientation exhibited less positive developments in self-efficacy (Peetsma & Van der Veen, 2013). Students who were twelve to thirteen-year-old students in the first and second year of the lowest level of secondary education (N=1,168), participated in the study. They were distinguished on avoidance-oriented students’ development in motivation for math, self-regulated learning behavior and achievement: Group 1 had an intermediate and stable performance-avoidance orientation; Group 2, a low and clearly increasing performance-avoidance orientation; Group 3, an intermediate and clearly decreasing performance-avoidance orientation; and Group 4, a low and slightly decreasing performance-avoidance orientation (Peetsma & Van der Veen, 2013). The content area focus of the study was mathematics. The participants were asked to focus on mathematics lessons when completing the questionnaires and the researchers
used the participants’ scores on mathematics as an indicator of achievement. The results indicated that groups showing less favorable trajectories in performance-avoidance orientation exhibited less positive developments in self-efficacy, well-being in school and performance-approach orientation.

**Inferences about Math-Related Motivation for the Study**

The middle-school years are critical time to identify students who are struggling academically and to intervene (Balfanz, 2009). The academic environment in middle school should feature nurturing and supportive interventions that can help direct students to success, especially students with LD (Anderman & Mueller, 2010). In the current educational context, more emphasis is placed on academic performance, often neglecting the important value of motivation, a significant predictor of academic achievement. It is imperative to explore the relationship between motivation and math achievement for middle school students with LD, especially since they often have deficiencies in number sense which may be related to decreased math-related motivation.

**Mindset**

*Definition of mindset.* According to Dweck (2006), students’ mindsets can be described in two ways: fixed mindset and growth mindset. A fixed mindset is a belief that basic abilities such as intelligence cannot be changed and are an inherent, stable feature that is not responsive to intervention. A student with a fixed mindset believes that abilities cannot be improved much, regardless of practice or effort. On the other hand, a growth mindset is a belief that the basic abilities can be developed through effort and repeated practice. A student with a growth mindset will often persist through their failures, believing that over time they learn and grow. Research has revealed that
mindsets can have a profound effect on achievement (Yeager & Dweck, 2012).

**Measurement of mindset.** Dweck’s Implicit Theory of Intelligence Questionnaire (Dweck, 2000) is used to measure fixed and growth mindsets. It includes eight items describing intelligence as being fixed or malleable. Participants complete the questionnaires and indicate their level of agreement on a 6-point scale (1=“strongly disagree” to 6=“strongly agree”). Fixed mindset items are reversed scored, and higher scores reflect a more growth-oriented mindset. The reliability of the mindset scale is well-established (De Castella & Byrne, 2015; Shen et al., 2016).

**Synthesis of Reviewed Mindset Literature**

Mindsets are related to academic achievement (Yeager & Dweck, 2012). Students can have different beliefs about their own intellectual abilities. Some believe that intellectual abilities are fixed, meaning they believe that abilities cannot be improved much with practice or effort, and some people believe that they can improve their abilities over time (Dweck, 1999). There are two implicit theories of intelligence: entity and incremental (Dweck, 2006). The entity theory focuses on measuring your ability, and everything that measures your ability. In contrary, the incremental theory focuses on learning and growth, and everything that can be helpful to learn and grow. These two implicit theories of intelligence shape the students’ goal, belief about effort, attributions for setbacks, and learning strategies (Yeager & Dweck, 2012). These theories are supported by numerous studies. For instance, seventh grade students in the incremental group showed significantly higher math and verbal achievement test scores, especially large effects were found among middle school girls in math (Good, Aronson, & Inzlicht, 2003).
Likewise, a significant effect of an incremental theory intervention was found for predominantly racial minority group of seventh grade students (Blackwell et al., 2007). The researchers found that mindsets can predict math and science achievement. Some of the minority public school students in New York City were making the difficult transition to 7th grade, and many were already showing declining grades, particularly in math (Blackwell et al., 2007). The students were divided into two groups, with each group receiving a six-session workshop. The control group’s workshop focused on study skills, but the students in the intervention workshop learned about both study skills and a growth mindset. The growth mindset intervention focused on how the brain grows new connections and gets smarter when a student works on challenging tasks. Results indicated that the math grades for students in the control group continued to decline. However, the students who received the incremental theory intervention, showed significantly greater improvement in math grades relative to the control group, with a significant effect of experimental condition on change in grades across the intervention (b = 0.53, t= 2.93, p <.05). The results supported that students with a growth mindset, compared to those with the fixed mindset, were significantly more oriented toward learning goals (Blackwell et al., 2007). Those with the growth mindset showed more mastery-oriented reactions to setbacks (Blackwell et al., 2007).

In another similar study, implicit theories interventions were effective for community college students with extremely high failure rate of math courses and placed in remedial math classes (Paunesku et al., 2015). It was found that about half of college students attend a community college in America (Kolesnikova, 2010), and about 65% of them place into math classes that are at the middle school or high school level, such as
arithmetic (Center for Community College Student Success, 2011). The results showed that whereas approximately 20% of students in the control group withdrew from their math classes, only about 9% in the incremental theory treatment group withdrew, which was a statistically significant difference (Paunesku et al., 2015). Of those who remained in the course, the students earned better grades and were less likely to fail (Paunesku et al., 2015). These intervention experiments were designed to change students’ mindsets and also to examine the effects on academic achievement.

A growth mindset of intelligence can impact academic achievement for students in secondary school (Dweck, 2006; McCuchen et al., 2016; O’Shea et al., 2010; Panunesku et al., 2015; Shen et al., 2016; Yeager et al., 2016; Yeager & Walton, 2011). A growth mindset intervention was found effective with over 1,500 high school students via digital mindset interventions (e.g., video available online) (Paunesku et al., 2015). The authors found a significant difference, in which lower-performing students benefitted most from the growth mindset intervention in terms of their GPA. A two-session growth mindset intervention for 9th grade students entering high school, developed through an iterative, user-centered design process, had an effect on their grades (Yeager et al., 2016). They also found that the intervention statistically significantly increased core course grades for previously low-achieving students. In addition, fixed mindset predominantly occurred among students across the high and low achieving students (Dweck, 2006). In one study, gender differences in mathematics performance only existed among fixed mindset students, suggesting that mindset is a critical contributor to math performance (Dweck et al., 2006). Fixed mindset beliefs can also contribute negatively in education, particularly harming students with learning disabilities (Dweck, 2006).
A significant relationship was found between goal-orientation, self-confidence, academic mindsets and persistence on math tasks (O’Shea et al., 2010). In this study, the researchers measured academic mindsets and persistence for challenging math tasks. They found students with a growth mindset reported persisting on challenging math tasks. Similarly, a significant effect of different mindsets was found on patterns of persistence when solving math problems with a group of college students ($n=137$) (Shen et al., 2016). The authors found that the students with more fixed mindset showed less persistence for high challenge tasks compared to low challenging tasks. There was also a significant relationship found between academic achievement and mindset when researchers compared students’ mindsets and their standardized test performance in math and reading over two years (grades 3-6, $N=419$) (McCuchen et al., 2016). The results showed that students’ initial mindsets can have an impact on their academic achievement. Students with a growth mindset had a slower decline on test scores than students with a more fixed mindset. These studies have demonstrated that mindsets can positively support effectiveness of educational environments and lead to long-term effects on students’ achievement (Yeager & Walton, 2011).

**Inferences about Mindset for the Study**

Research has shown that students’ own views on intelligence, fixed or growth, influence how they respond to academic challenges (Blackwell et al., 2007). Studies have also provided convincing evidence that mindset and achievement are significantly related. Students with LD often tend to have a more fixed mindset compared to their typical peers (Hartmann, 2013). A history of academic challenges, frequent difficulty and failure may contribute to a propensity toward more fixed mindsets. Furthermore, mindset
is directly related to motivation (Chirila & Ticu, 2016) and students with disabilities are at risk for decreased motivation in conjunction with fixed mindsets (Morgan et al., 2008). Recent work has clearly indicated that mindset is malleable and can be learned through targeted intervention. Do growth mindset interventions improve math-related motivation for students with learning disabilities? And would increased growth mindset and motivation improve math achievement and/or number sense? It is imperative to explore the relationship between these variables and math achievement for middle school students with LD, to inform the importance of effective intervention at a critical time for those students.

**Strengths and Limitations of Extant Literature**

There were several strengths evident in the prior studies. First, the inclusion of different methodologies has allowed for a multidimensional exploration of the topic. Second, these studies included a wide range of participants, from infants to adults. Even though most of them focused on the younger children, the consistent results from the wide range of the ages represented reflect the overall experiences of the population in general. A third strength is that the studies have investigated the constructs in multidimensional ways. Not only they have analyzed each construct, but also explored it with diverse perspectives of cognitive, emotional, and motivational lenses.

While the literature reflects some strengths, there are also significant gaps. First, although the studies included a range of participant ages, they did not explicitly focus on older students. Additionally, more studies are needed describing certain populations in terms of middle school students with LD. The cognitive, emotional, and motivational constructs were found to be significantly related to math achievement. However, the
above literatures fail to comprehensively examine middle school students with LD. Third, findings are not relevant to understanding math achievement for middle school students with LD in consideration of mindset, motivation, and number sense. No one has investigated at these constructs together for this specific population.

**Theoretical or Conceptual Framework for The Study**

Improving the math achievement of students with LD has become an important goal for schools (IDEA, 2004). Despite the investments in education reform such as IDEA, students with disabilities continue to exhibit much poorer math performance compared to their peers without disabilities (NAEP, 2015). This poor achievement demonstrated by students with LD can be attributed to many factors, including past failure experiences, low expectations, and learned helplessness (Ju, Zhang, & Katsiyannis, 2012). These factors can shape how students view their competence and can also affect their academic achievement (Ju et al., 2012).

The factors impacting math achievement are complex. One well-established correlate of math achievement is number sense. Number sense as a foundational skill may also be a facilitator that allows students to develop strategies for solving complex math problems. Number sense is very complex and multifaceted in nature. It comprises a deep understanding of a wide range of concepts in numbers. In general, it involves awareness, intuition, recognition, knowledge, desire, conceptual structure of numbers (Berch, 2009). Number sense reflects the ability to use numbers to communicate and interpret information (McIntosh et al., 1992). Some consider number sense as an acquired skill through experience, while some think of number sense as an inherent quality (Berch, 2009). It can be a construct that may respond to environmental exposure and learning
In this study, number sense will be defined as, “critical thinking applied to a quantitative context” (Gittens, 2015, pg. 2). It reflects the ability to make judgments based on quantitative information in a variety of contexts.

Additionally, research has shown that self-concept is significantly related to academic achievement for students with LD (Meltzer et al., 2004; Zheng, Erickson, Kingston, & Noonan, 2014). There is an evidence that students with LD who had less negative perceptions of their disability showed higher math achievement scores and more positive global self-concept (Zeleke, 2004). Thus, identity formation can affect student educational experience and performance (Lau & Roeser, 2002). A mathematics self-concept has been found to have a significant effect on number sense (Geronime, 2002). This self-concept can be developed by self-evaluations, and it contributes to perceptions of oneself (Marsh, Xu, & Martin, 2012). According to Marsh (1986), self-concept can be shaped by students’ comparison of their performances to those of others and also comparison in their performance in subject areas. Likewise, students’ values of mathematics can reflect their propensity to pursue academic challenges in math. This effect of math self-concept on number sense augments the link between cognitive and emotional and motivational construct. It suggests that student’s beliefs and interest in mathematics can contribute to math achievement. Therefore, a lack of number sense, a definitively cognitive construct, may interact with motivation and/or mindset to have a significant effect on math achievement in students with LD. Motivation and mindset are interrelated constructs in which student’s perceptions can lead to self-evaluations and task-persistence (Chirila & Ticu, 2016).
Motivation, also a complex and multifaceted construct, can be described in general, as the arousal, direction, and persistence of behavior (Franken, 2006). Motivation constitutes maintaining attention to something interesting, developing meaning, or solving a problem (Huitt, 2001). It also involves maintaining levels of optimism or developing self-efficacy (Huitt, 2011). The regulation of academic achievement motivation can depend on various beliefs systems held by students. For example, these motivational beliefs include: goal orientation (purpose of the task), self-efficacy (judgments of competence), perceptions of task difficulty, task value beliefs (beliefs about the importance), and personal interest in the task (liking of content area) (Pintrich & Schunk, 2002). These beliefs are important in self-regulation of motivation that can influence achievement and learning. Motivation can thus reputedly mediate the relationship between the person and the context in learning (Rotgans & Schmidt, 2012). Thus, there are potential cognitive and emotional facets of motivation relative to academic motivation that can contribute to math achievement in students with LD.

Students' academic self-concept can strongly influence their academic self-efficacy beliefs, and academic self-concept is a predictor of affective–motivational variables (Ferla, Valcke, & Cai, 2009). In addition, achievement, self-efficacy and self-concept are strongly correlated in mathematics (Parker et al., 2014). Educators have long recognized that students’ beliefs about their academic capabilities play an essential role in their motivation to achieve, but such self-conceptions regarding academic performance are complex to define (Zimmerman, 2000).

The conceptual difference between self-concept and self-efficacy beliefs is not always clear. Ample empirical evidence supports that self-concept and self-efficacy are
related and can influence academic achievement by influencing effort, persistence, and perseverance (Zimmerman, 2000). Self-efficacy can be related to both motivation and mindset. Self-efficacy reflects one’s judgments of the capability to do the work and one’s perception of internal control over their own learning and effort (Schunk & Ertmer, 2000). It influences the choices that people make and the courses of action that they pursue, and the level of accomplishment that individual ultimately realizes (Schunk & Ertmer, 2000).

Mindsets can have an impact on students' sense of control over their learning, while self-efficacy reinforces the value of it (Schunk & Ertmer, 2000). Academic mindsets play an important role in building engagement and resilience. Growth mindset refers to the belief that effort and perseverance can contribute to better performance (Dweck, 2007). It builds a cognition that intelligence can be cultivated, not fixed. With growth mindset, abilities are related to the habits of mind. Students with a growth mindset can attribute success to persistence and practice and will place more emphasis on the learning process (Paunesku et al., 2015). These beliefs can influence the emotions that students experience as they encounter challenges. Thus, mindset can have substantial impact on the student’s performance. The cognitive, emotional, motivational components of mindset can contribute to math achievement in students with LD.

There is a rapidly growing number of neuroscience studies on cognitive functions. There is evidence of neuroscientific interplay between growth mindset and intrinsic motivation (Holroyd, & Yeung, 2012; Moser et al., 2011). Neuroscience research aims to better understand the biological basis of emotional processing. A proliferation of affective neuroscience literature has shown that emotion and cognition are not separate constructs (LaBar & Cabeza, 2006; Ochsner & Gross, 2008; Steinberg, 2005). Studies
have explored cognitive processes in emotion, and the processes and anatomical structures involved in emotion (Kohn et al., 2014; Kringelbach, & Berridge, 2017). Thus, it is important to understand the complex interactions of these variables to math achievement, especially in students with LD who are at high risk to struggle in math.

The empirical work at hand will inform unresolved questions: How do number sense, motivation and mindset relate to one another and contribute to math achievement in middle school students with LD? Number sense is a complex yet important concept in the educational field. Students need a strong foundation of number sense in order to successfully comprehend mathematics (Ivrendi, 2011). Researchers, in general, claim that number sense can be defined as critical thinking applied to a quantitative context, deeply understanding the meaning of numbers (Gittens, 2015). In addition, affective variables are salient factors affecting success and persistence in math (Singh et al., 2002). Previous affective neuroscience literature has shown that emotion and cognition are not separate constructs. Also, mindset and motivation are interrelated constructs in which student’s perceptions can lead to self-evaluation and task persistence (LaBar & Cabeza, 2006; Ochsner & Gross, 2008; Steinberg, 2005). Yet the available research evidence does not conclusively support the claims that this group of variables (number sense, mindset, and motivation) are related to math achievement concurrently for middle students with LD.

The proposed study will examine the hypothesis that number sense, motivation and mindset are related to each other and have a significant effect on math achievement (see Figure 1). A hypothesis will be tested to see if number sense, motivation and growth mindset are significantly lower in students with LD when compared to their peers without disabilities. To examine the validity of the IA numeracy scale (Gittens, 2015), a
hypothesis will be tested to compare teachers’ evaluations of student’s number sense to the scores on the IA numeracy scale (Gittens, 2015). In addition, the perceptions of the students with and without disabilities on mindset and motivation will be analyzed to explore their experiences with and achievement in math. Their positive and negative experiences with math achievement will be also evaluated.

![Diagram](image)

*Figure 1.* The hypothesized relationships between mindset, number sense, and motivation which are thought to be different based on disability status to math achievement.
CHAPTER 3: METHODOLOGY

Introduction

There is ample research evidence to suggest that number sense, motivation, and mindset represent cognitive, emotional, and motivational variables that are correlated to math achievement. However, previous research has not examined the role of these constructs for middle school students with learning disabilities (LD). The current study has two primary purposes. The first purpose of this study is to explore relationships between math achievement and three variables: mindset, motivation, and number sense for middle school students with and without LD. The focus is to explore how these cognitive (number sense), emotional, and motivational (mindset and motivation) components contribute to variance in math achievement among students with and without LD. The second study purpose is to explore middle schoolers’ perceptions of motivation and mindset factors on their math achievement, as well as explore how they explain other potential factors related to their positive and/or negative experiences in math.

Research Questions

The following research questions were investigated through collecting both quantitative survey data and qualitative interview data and analyzing data statistically, coding for themes, and integrating findings from both quantitative and qualitative data.

1. Do scores on measures of each of the three independent variables (number sense, math motivation, and mindset) differ between middle school students with LD and students without LD?

2. Do number sense, math motivation, and mindset predict the dependent variable of math achievement in middle school students?
a. After controlling for disability status, are there differences in math achievement based on number sense, mindset, and motivation?

b. After controlling for number sense, are there differences in math achievement based on mindset and motivation?

3. Do teacher evaluations of student number sense correlate to scores on the IA numeracy scale (Gittens, 2015)?

4. How do middle school students with and without disabilities perceive mindset and motivation to relate to their experiences with and achievement in math?

5. How do middle school students perceive positive and negative experiences with achievement in math?

**Hypotheses**

These research questions were developed to explore these cognitive, emotional, and motivational variables in relation to math achievement for middle school students. I conducted this study with the following assumptions:

**Hypothesis 1:** There would be strong associations between the predictors (number sense, motivation, and mindset).

**Hypothesis 2:** There would be significant differences on scores on measures of these constructs between the two groups (students with LD and those without). I predicted that the results would reveal that the group of students with LD would score lower on all measures, potentially suggesting the need for support during this transitional period before high school.

**Hypothesis 3:** There would be no significant effect of mindset and motivation on math achievement after controlling for number sense.
**Hypothesis 4:** The scores on the IA numeracy scale (Gittens, 2015) would be strongly correlated with the teacher evaluations of student number sense.

**Hypothesis 5:** Compared to students with LD, students without LD would report more positive mindset and motivation toward their experiences with and achievement in math.

**Overview of Methodology**

In this study, I used a mixed method to explore variables related to math achievement. Qualitative and quantitative methods were implemented concurrently. Quantitative methodologies can address questions about generalizability or magnitudes of effects, while qualitative methodologies can explore the nature of an individual’s experiences (Fetters et al., 2013). A mixed methodology draws upon strengths of both approaches. The integration of quantitative and qualitative methodologies enhances the advantages of mixed methods research (Creswell, Plano, & Clark, 2011). Quantitative data can help explain findings from the qualitative data, and qualitative data can provide context regarding the validity of quantitative findings (Fetters et al., 2013).

A quantitative approach has many advantages, such as accurate measurement of specific constructs, the capacity to conduct group comparisons, and the ability to examine associations between variables (Castro, Kellison, Boyd, & Kopak, 2010). However, there is a limitation of the possibility of detaching information from its original “real-world.” The quantitative data may not fully capture and reflect the real-world context completely. A qualitative approach can overcome this limitation. It generates rich detailed accounts of human experiences, and the narrative accounts can be examined within the original context (Castro et al., 2010). Thus, it affords an in-depth analysis of complex experiences
in a manner that cannot be fully captured with a quantitative-only approach. Qualitative methods can attempt to answer how or why a phenomenon occurs within complex contexts, where variables are difficult to measure (Trainor & Graue, 2014). A basic interpretative approach to qualitative research allows the researcher to be the key instrument in the inquiry, in which the researcher makes judgments on the processes of analysis (Trainor & Graue, 2014). It is constructive to investigate how students make meaning of the situation and to understand the meaning of the experiences with the qualitative method. The analysis of themes and patterns can provide rich descriptive accounts from participants (Merriam et al., 2002).

This study used a mixed method in a convergent design framework, where both quantitative and qualitative data was collected during a similar time frame (Fetters et al., 2013). Qualitative and quantitative data collection occurred in parallel; that is, the data collection occurred at the same time for the baseline survey and interviews of a subsample of participants. After the completion of data collection, both qualitative and quantitative data was analyzed separately, then merged. The integration of the data occurred through joint displays to simultaneously display the quantitative and qualitative results (Gutterman et al., 2015). A joint display connects findings back to the theoretical framework of the study (Gutterman et al., 2015). It examines both type of data (quantitative and qualitative) and also allows for them to be examined side by side for new insights (Gutterman et al., 2015). This integrated analysis provided meaningful insights into relationships between number sense, motivation, mindset, math achievement, and disability. The fit of the integration was examined for both confirmation, in which the both types of data confirm the results of the other, or
expansion in which the two sets of data diverge and provide new insights from the results (Fetters et al., 2013).

Three statistical analyses were used in effort to answer the research questions: correlations, ANOVA, and ANCOVA. To examine how teacher evaluations and scores on the IA scale are related, I ran a correlation analysis. An ANOVA was used to determine whether there were any statistically significant differences between the means of the groups in relation to math achievement and also to determine whether the influence of number sense, math motivation, and mindset on the math achievement was present. I also used ANCOVA (analysis of covariance) to investigate whether the ability of these variables to predict math achievement differs between groups. The statistical control of ANCOVA helped explain the variation in the dependent variable when controlling for selected covariates. The effects of the covariate were adjusted for math achievement. ANCOVA statistically controlled the effect of one or more variables by removing the effect of covariates. I conducted ANCOVA to find what effect an independent variable had after the effect of the covariate was accounted for. Number sense and disability status were covariates for the analysis.

Sample sizes for ANOVA and ANCOVA were determined using the power analysis tool, Gpower Calculator. I conducted a range of power analyses, using an alpha of 0.05, a medium effect size ($f = 0.25$), with a power of 0.80 and a power of 0.95. For ANOVA, the desired sample size was 159 (power of .80) and 252 (power of .95). For ANCOVA, the desired sample size was 128 (power of .80) and 210 (power of .95). I also conducted the power analysis using a large effect size ($f=0.4$). For ANOVA, the desired sample size was 66 (power of .80) and 102 (power of .95). For ANCOVA, the desired
sample size was 52 (power of 0.80) and 84 (power of .95). Due to limited sample size, there may be challenges in detecting group differences, especially small to medium ones, due to lack of power in the present study. Thus, I also used collected qualitative data and integrated both sets of data to gain a deeper understanding of the relationships between mindset, motivation and number sense with math achievement in middle school students with and without LD.

**Research Procedure**

**Setting**

The study sample was recruited from an urban middle school in Indianapolis, Indiana. The convenience sample was recruited at a public charter school, which serves 698 students in grades K-8. Of those students, 196 students are in grades six through eight. The school serves students both with and without special education needs. The school uses ISTEP+ (Indiana Statewide Testing for Educational Progress Plus) to ensure that instruction is differentiated to varying student ability levels, and that student growth is adequately monitored. The school is also considered a Title I school, indicating the relatively high proportion of low-income children in the district (school’s poverty rate is 40% or higher). The K-8 school population has a relatively high percentage population of African-American students (47%), with 27% identifying as Caucasian, 16% as Hispanic, and 9% as Multiracial. For middle school (grades six through eight), the percentages are similar (Black: 44%, White: 30%, Hispanic: 17%, and Multiracial: 8%).

The middle school at this particular school integrates a looping structure, in which the teachers remain with the same group of students for three years (sixth through eighth grades). There are two math teachers, two reading/writing teachers, and one science
teacher. As students enter middle school, they have the same teachers for three years until they graduate to high school. The special education teacher also works with sixth through eighth grade students. All classes are inclusive. The special education teacher pushes in or pulls out to work in small groups as needed.

**Sampling Procedure**

The sample for this study included 6th, 7th, and 8th grade students in a public charter school. There were two groups of middle school students who were recruited to participate in the study: students with LD and students without LD. There were approximately 40 students with LD in this school. The goal for the sample size for each group was a minimum of 30, but due to incomplete questionnaires and unavailability of past ISTEP scores for some students, the final sample was comprised of 48 students. Working together with the support and partnership of the school, students were alerted to the opportunity to participate through flyers disseminated at school, social media posts, and word-of-mouth. In addition to those means, parents were alerted via an email sent from the school administration which contained consent and assent materials. Parental consent and student assent were collected prior to gathering data. There was an IRB-approved participation incentive to maximize participation. Once the parents have signed the consent forms and students the assent forms, forms were collected by me and stored in a secure location.

**Participants**

Participants were students in sixth through eighth grade and included students with and without LD. Students were considered to have LD if their current IEP contained
a diagnosis of LD. Students were considered to not have a disability if they did not have an IEP or a 504 plan.

Measures

Qualitative and quantitative data were collected at the same time and analyzed independently (Creswell et al., 2003). Valid and reliable tools with acceptable psychometrics, were utilized to gather quantitative data on motivation, mindset, and math achievement. The IA Scale (Gittens, 2015) was used to measure number sense, along with teacher evaluations of the student’s number sense. The standardized state test scores were used to measure math achievement. Motivation and mindset were measured with both quantitative and qualitative data: surveys were used to collect quantitative data and I collected qualitative data on motivation and mindset through 1:1 interviews with students. The measures are described in detail below and were administered in school over spring and/or summer months.

Number sense. To measure number sense, the IA Numeracy Scale (Gittens, 2015) was used. The fundamental priority in developing the IA scale was to assess skills of analysis, inference, interpretation, explanation, and evaluation, with no items that rely on mathematical computations. The assessment conceptualizes number sense as critical thinking relative to quantitative information. The IA Scale includes two levels, which are based on the typical developmental span characteristics of children and adolescents within the targeted grade-level ranges (grades 3-5 and grades 6-8) (Gittens, 2015). To ensure developmentally appropriate mathematical content, the developers mapped the elements of the assessment to the draft Common Core Standards for Mathematics. Items on the numeracy scale required respondents to demonstrate understanding of quantitative
information by visual representations in real world contexts. The measure consists of 11 items for sixth through eighth grade students (KR-20=.69). The processes used to evaluate its reliability and validity are consistent with the Standards for Educational and Psychological Testing (AERA, APA, & NCME, 2014). The construct validity of the IA numeracy scale was examined with respect to the published measures of mathematical achievement. The results of this study indicated that the numeracy scores were positively correlated with the ITBS math total score, correlated at $r=.574$ (Gittens, 2015). In addition, students’ numeracy scores on the IA numeracy scale were also strongly and positively associated with the CogAT composite score. These results indicated that 40 - 56% of the variance in the CogAT composite scores could be explained by scores on the numeracy scale (Gittens, 2015).

In addition to the IA numeracy scale, I collected teachers’ evaluations of student’s number sense. Using Gitten’s definition of numeracy, “critical thinking applied to a quantitative context” (Gittens, 2015, p.2), teachers rated the student’s number sense from one through five (1=low, 2=moderately low, 3=average, 4=moderately high, 5=high). The results were compared to the scores on the IA numeracy scale (Gittens, 2015). A statistically significant correlation reflected the validity of the IA numeracy scale (Gittens, 2015) (discussed further below).

**Motivation.** Student motivation data was collected via surveys administered in school. The surveys were adopted from previous study on student motivation for mathematics with 8th graders (Simzar et al., 2016). Motivation questionnaires were based on two widely-used frameworks: achievement goal theory and expectancy-value theory. The motivational measures were achievement goals, self-efficacy, and subjective task
value. The achievement goals were measured using three scales from the Patterns of Adaptive Learning Survey (PALS; Midgley et al., 2000; Midgley et al., 1998). All items were assessed using a 5-point Likert scale ranging from 1 (not at all true for me) to 5 (very true for me). The achievement goals include mastery goals, performance-approach goals, and performance-avoidance goals. Mastery goals were assessed using items that focus on learning and understanding. Performance-approach goals include items that focus on demonstrating ability and outperforming others. Performance-avoidance goals focus on not appearing incompetent. In addition, students’ beliefs in their abilities were measured with the Academic Efficacy scale from PALS (Midgley et al., 2000). Self-efficacy assessed students’ judgments about their ability and confidence to perform adequately in math. All items were assessed using a 5-point Likert scale. Value was assessed with four scales adapted from the work of Eccles, Wigfield, Blumenfeld and their colleagues (Eccles et al., 1993; Eccles & Wigfield, 1995; Wigfield et al., 1997). The four subscales measured: interest, utility attainment, and cost. Interest was assessed using items that focused on students’ attraction to, liking for, and enjoyment of math. Attainment was assessed using items that focused on students’ judgments about the importance of math for their sense of who they are. Cost was assessed using items that focused on students’ judgments about the amount of time and effort required to be successful in math. The survey includes a total of 41 items. I used Mastery Goal (5 items, α = .81) and Academic Self-Efficacy items for math (4 items, α = .85) for the math motivation survey (n=9 for all items). These items were used to examine how much students focus on learning and understanding in math and also their enjoyment of math. The confirmatory factor analysis demonstrated high factor loadings. The standardized
factor loadings and internal consistencies for these constructs ranged from acceptable to good indicating acceptable construct validity (all standardized factor loadings are significant at \( p < .001 \)). Chi-square = 3,883, df = 601, \( p < .001 \); CFI = 0.95; TLI = 0.95; RMSEA = 0.04; SRMR = .04.; Midgley et al., 2000). See Scale in Appendix A.

**Mindset.** To measure mindset, Dweck’s Implicit Theories of Intelligence Scale was used (Dweck, 1999). The survey consisted of eight questions answered on a 6-point Likert type scale. This scale asked students to indicate the extent to which they agree or disagree with statements pertaining to growth or fixed mindset. The scale contained four items measuring “fixed” mindset factor (\( \alpha = .77 \)) and four items for “growth” mindset factor (\( \alpha = .83; \) De Castella & Byrne, 2015). The scale has been found to have good construct validity with scores predicting theoretically meaningful relationships with a range of variables (Dweck et al., 1995). A confirmatory factor analysis (CFA) indicated that this model represented an acceptable fit for the proposed structure of the scale, \( \chi^2(37, N=481) = 114.07, p < .05, \chi^2/N = .23; \) RMSEA = .07, GFI = 0.96, NFI = 0.96, CFI = 0.97, IFI = 0.97, RFI = 0.94 (Shih, 2011). Individual student totals will be tabulated to indicate their mindset scores. See scale in Appendix A.

**Math achievement.** The students’ math scores were collected from the district annual testing records, Indiana Statewide Testing for Educational Progress Plus (ISTEP+). The purpose of ISTEP+ is to measure student achievement in the subject areas of English/Language Arts, Mathematics, Science (Grades 4, 6 and 10), and Social Studies (Grades 5 and 7). ISTEP+ reports student achievement levels according to the Indiana Academic Standards, adopted by the Indiana State Board of Education. The assessments consist of two parts: Part 1 consists of applied skills (open-ended) items, and Part 2
includes multiple-choice and technology-enhanced items. Both Part 1 and Part 2 are used to measure student mastery of the Indiana Academic Standards (Indiana Department of Education, 2017). ISTEP+ is a statewide test and analyses from the state of Indiana support the validity and reliability of the test. Cohen’s kappa for the math tests is an average of approximately 0.80 at the Pass level and approximately 0.75 at the Pass+ cut. For the ISTEP+ tests, the classification accuracy is approximately 0.90 with a range between 0.87 and 0.93 for the spring 2016 administration. The internal consistency estimates are approximately 0.95 for math (IDOE, 2015). There are three critical validity claims for the ISTEP+: Systems Coherence, Comparability or Procedural Quality, and Accessibility and Fairness. The test was designed to yield scores that reflect students’ knowledge and skills in relation to academic expectations. The test is also focused on the aspects of the administration and procedural analyses and reporting structures. It was designed so that the students take the assessment under conditions that allow them to demonstrate what they know and can do in relation to academic expectations. It was designed to ensure the production of high-quality assessment instruments that accurately measure the achievement of students with respect to the knowledge, skills, and abilities contained within the Indiana Academic Standards (IDOE, 2016).

ISTEP is a criterion-referenced test. It consists of items that assess a student’s performance with respect to the Indiana Academic Standards established by the Indiana State Board of Education (IDOE, 2016). The test does not provide norm-referenced information. The performance of Indiana students is not compared with that of students across the nation. Criterion-referenced scores indicate where a student stands in relation to the cut score, defined by educators, and based on Indiana Academic Standards (IDOE,
A student’s score is developed based on Item Response Theory, which analyzes the data obtained from test questions. IRT is used to determine how difficult each item is and how well each item distinguishes students who do and do not have the skill being tested by the item (IDOE, 2016). The ISTEP+ test scale is divided into three performance levels using two cut scores. The cut score is the score that separates three performance levels: Pass+, Pass, or Did Not Pass (IDOE, 2016).

**Interviews.** All participants were assigned a participant ID number as a unique identifier. A minimum of six students from each group (LD or non-LD) were selected to participate in the interview. The interviews were based on a semi-structured interview protocol, in which students were asked similar questions and their responses were audio-recorded and transcribed within one week of interview. Careful notes were also kept during the interview in case of technology failure. Interviews were conducted in the school building, in-person. The questions were guided by the semi-structured interview protocol (see Appendix A).

In addition, I recruited one middle school math teacher and one special education teacher for the number sense evaluation. The students see each teacher daily due to their schedules (two Math classes, two Reading/Writing classes, and one Science). Both teachers have been working with the students and have a good understanding of them. Before administering the numeracy scale, the teachers were interviewed for their evaluations of students’ number sense (see Appendix A). They rated student’s number sense based on Gitten’s definition of numeracy, “critical thinking applied to a quantitative context” (Gittens, 2015, p.2). They rated on a scale of one through five (1=low, 2=moderately low, 3=average, 4=moderately high, 5=high). I, as one of their
math teachers, also participated in evaluating the students’ number sense. The mean of the responses was correlated to the scores on the IA numeracy scale (Gittens, 2015).

**Procedure**

**Step 1.** Upon obtaining approval from IRB, the principal was notified about the details of the process. I worked closely with the school to set an appropriate time for the study, to minimize the loss of academic time during the course of the study.

**Step 2.** Next, I recruited participants by disseminating flyers to the students and to parents, via email, about the study. I utilized the school’s available social media platform to advertise too, as well as used word-of-mouth. I provided consent and assent materials via email and hardcopy, since participants were younger than 18 years old and needed parent consent for the study. The students received incentives for submitting the forms, as negotiated with the school and approved by IRB. In the recruitment letters, the parents were notified of the research procedures, including the purpose, assessments, incentives, and confidentiality.

**Step 3.** After the consent and assent materials were collected, the letters were scanned electronically and saved in a file encrypted with a password. This protected confidentiality and also secured the information. The papers were shredded after the transfer as electronic files.

**Step 4.** Each student was given a participant ID number to protect their identity since I linked their interview data to their survey data for the integrated analysis.

**Step 5.** Two teachers (a math teacher and a special education teacher) were recruited to evaluate each student’s number sense. I also evaluated students’ number sense as their math teacher. Scores were entered on Microsoft Excel spreadsheets and
saved it in a file encrypted with a password. To assure accuracy, teachers repeated their answers as they were recorded. I also entered my own evaluations on Microsoft Excel spreadsheets and saved as encrypted files.

**Step 6.** On prescheduled days, the surveys were administered in an assigned room. The students received a pass to come from their teachers prior to the scheduled time for instructions. Participants completed the IA numeracy scale (Gittens, 2015), the motivation scale and the mindset scale. The approximate time for them to complete the surveys were 40-50 minutes. The IA numeracy scale (Gittens, 2015) was online, and students utilized school-provided iPads. I received and stored the scores electronically. The scores were linked to Participant ID and secured in a password encrypted file to protect their privacy. Students then completed the motivation and mindset scales via an online survey delivered though the Qualtrics © platform. Again, students were identified only by participant ID and scores were stored in an encrypted file by Participant ID only.

**Step 7.** The students were interviewed one at a time individually. All interviews were recorded using a digital recording device. The school has a record of students with media permission and parents consented to this again the study consent paperwork. Only the students who had the media permission were recorded. For students who did not have media permission, their answers were recorded through typing instead. The completed interviews were saved on a password encrypted file. I transcribed the interviews and analyzed them with the QDA Miner software.

**Step 8.** The approximate time to complete the questionnaires and interviews was in the span of two weeks. For the students who were absent on the survey or interview days, they did a make-up on another day individually. The participants received a
participation incentive, approved by the school and IRB, for completing the surveys and an additional incentive if selected for the interview.

**Step 9.** To obtain math achievement scores, the ISTEP + scores were collected from the academic record from the previous year. The records were available electronically. The scores were recorded electronically in an Excel file and linked by participant ID number to preserve confidentiality.

**Step 10.** The collected quantitative data was entered into SPSS for analysis. It is critical to enter correct numbers, thus the data was reviewed and double-checked to assure accuracy.

**Analysis**

The analysis of both qualitative and quantitative data was conducted separately and then the findings were subsequently merged for additional analysis. The results were compared and synthesized to enhance understandings of the variables associated with math achievement for middle school students. This study included an integrated convergent analysis. The quantitative data was used to further examine the qualitative data, and vice versa.

**Screening process: Descriptives.** Quantitative data was analyzed with descriptive statistics, which included: percentages, means, standard deviations, and correlations. I reported percentages for the participants’ demographic data, including their grade levels. I also reported means and standard deviations for participants’ scores on measures of number sense, motivation, mindset, and math achievement. Additionally, I calculated Pearson’s correlation coefficients for the variables (number sense, motivation, mindset, and math achievement). The descriptive statistics for all variables and
correlations served to screen the data for assumptions of ANCOVA. The descriptive statistics were conducted in SPSS.

**Analysis 1: Interrater reliability.** The evaluations of number sense by three teachers were collected. The mean of the scores was calculated for each student. The results were correlated to the scores on the IA numeracy scale (Gittens, 2015). An interrater reliability (IRR) analysis was used to examine whether the teacher scores correlated with one another and also to examine whether they correlated to the IA numeracy scale (Gittens, 2015). IRR is a statistical measurement that can determine how similar the data collected by different raters are and the level of agreement between raters. The closer IRR is to 1, the closer the agreements are. It also provided additional data to triangulate with the IA numeracy scale (Gittens, 2015).

**Analysis 2: ANOVA.** Analysis of variance (ANOVA) was used to analyze factors that effect a given data set. It evaluates the impact of a sole factor on a sole response variable and determines whether the samples are the same. The ANOVA was used to determine whether there were statistically significant differences between the means of the groups (student with and without LD) in relation to math achievement. The F- Ratio was calculated to measure if the means of different samples were significantly different or not. The lower the F-Ratio, the more similar the sample means.

**Analysis 3: ANCOVA.** Analysis of covariance (ANCOVA) was used to eliminate the effects of a covariate from the relationship between the dependent variable and independent variable and examine whether there are meaningful relationships among variables after the removal of the covariate effects (Tabachnick & Fidell, 2013). To study how the cognitive, emotional, motivational components of mindset, motivation and
number sense are related to math achievement for students with and without learning disabilities, ANCOVA was conducted on the collected data. ANCOVA analysis was used to determine the extent to which there is a linear relationship between a dependent variable and the independent variables with covariate(s) (Tabachnick & Fidell, 2013). The purpose of the ANCOVA was to explain the effects of selected primary independent variables (motivation, and mindset) on math achievement, while removing the effects of the covariate (number sense). Therefore, I assumed that this overall relationship is true for all groups of participants. I screened which predictors were reasonably correlated with the dependent variable. I used ANCOVA to increase power and examine the effects of the variables. I planned to assign number sense as covariate; however, if there was no significant correlation for number sense from the screening analysis, I planned to use another covariate (based on the correlation results among variables). Assigning covariates reduced the probability of Type II error when analyzing the interaction effects.

The ANCOVA F test evaluated whether the population means on the students’ math achievement, adjusted for differences on the covariate, was different across variables: \( F = \frac{MS_p}{MS_W} \). I ran ANCOVA with assumptions for a linear relationship between dependent variable and the covariate and homogeneity of regression for adjusting sample means (Tabachnick & Fidell, 2013). In addition, there were three other assumptions underlying ANCOVA: 1) assumption of independence (the scores on the dependent variable are independent of each other) 2) assumption of normality (dependent variable is normally distributed), and 3) assumption of homogeneity of variance (variances for conditional distributions are equal) (Green & Salkind, 2003). To compute the effect size for ANCOVA, Cohen’s \( d \) was used. ANCOVA was conducted in SPSS.
Analysis 4: Qualitative analysis. The qualitative data from the interviews was analyzed through two cycles of coding. The first cycle of coding was open coding. To build concepts from the interview, I used open coding to uncover the meaning, ideas and thoughts embedded in the texts. With open coding, I was able to label concepts and develop categories based on their properties. I attempted to turn the data into concepts by attaching labels to the words that represented an interpretation of them with open coding (Given, 2008).

To make connections to my study, I took analytic memos through first and second cycle coding to identify patterns. The second cycle involved analyzing data from the first cycle. The second cycle filtered and highlighted data to generate themes and concepts of the study (Saldana, 2013). I anticipated using focused coding to organize and streamline codes from my first cycle coding. I used QDA Miner software to analyze data. I used a variable-by-variable matrix in the second cycle of coding which was useful for finding relationships among variables.

Analysis 5: Integrated convergent analysis. The results of both qualitative and quantitative data were compared and synthesized to enhance understandings of the variables associated with math achievement for middle school students. This study included an integrated convergent analysis. The quantitative data was used to further examine the qualitative data, and vice versa. Both quantitative and qualitative data were collected during a similar time frame. The analysis for integration occurred through connecting method of data collection and analysis. Connecting method linked one type of data with the other through the sampling frame. Both sets of data were merged for analysis. Integration through a narrative with weaving approach stated both quantitative
and qualitative findings together on a concept-by-concept basis (Fetters et al., 2013). To explore relationships between the quantitative data and generated themes from the qualitative data, I used joint displays, as a cognitive framework, to simultaneously display the quantitative and qualitative results (Gutterman et al., 2015). A joint display provides a visual means to integrate the data. It can validate quantitative knowledge while developing qualitative knowledge (Gutterman et al., 2015). A convergent design with a merging approach to integration represented results with a themes-by-statistics display (Gutterman et al., 2015). This integrated analysis provided meaningful insights into relationships between number sense, motivation, mindset, math achievement, and disability. It both confirmed data collected and expanded understandings in new directions. I also used QDA Miner for analysis of integrated matrix (Greenacre, 1993).

**Researcher Reflexivity**

In this study, I interviewed a selected sample of middle school students. I work closely with the students as a teacher. There was a possibility for me to influence the responses of the students. I attempted to reduce possible threats to validity of the qualitative research, most notably bias and reactivity (Maxwell, 2013).

In order to accurately interpret the experiences of the students, I had to first consider my own experiences and bias to be mindful of my interpretation. From my experiences of teaching math to middle school students with LD, I have seen students struggle and eventually lose interest in math. The gaps seem to widen as they struggle to acquire more complex concepts in math. I have seen the repeated failures to comprehend the materials exhaust the students. The students subsequently believed that they needed help before attempting math work independently. These signs of gradual learned...
helplessness alerted me to seek a deeper understanding of the scholarly literature to help the students. I started my doctoral studies with a goal of improving education for students with LD, as I had watched my own students with histories of failure in math become helpless and disengaged. As a result, I worked hard to be diligent to put aside my biases and manage subjectivity to increase validity for the present study. Especially when I evaluated the student’s number sense as a teacher, I truthfully rated the students based on their performance and experiences. I delivered the accuracy and truth of the study to provide meaningful research.

In addition, the interview process involved interactions with the students. Students’ perceptions and responses were meaningful for data collection. I have already established relationships with the students. This established rapport helped me conduct research with a level of trust, which helped the students feel comfortable and led to rich data for meaningful analysis. I also anticipated the possibility that the students might have difficulty focusing or effectively expressing their thoughts. I recognized these possibilities and used caution in development of the interview protocol, and generated thoughtful engagement with the students by making them feel comfortable with a conversational style interview.

**Human Participants and Ethical Considerations**

This study facilitated human subjects research and ensured the rights and welfare of human subjects during participation, guided by the Institutional Review Board (IRB). I provided the opportunity to consent or decline to participate to the students’ parents. I also procured student assent to participate. I carefully reflected on my choice of words to represent the data truthfully. I recognized the vulnerability of the sample population and
took precautions to protect participant’s confidentiality, privacy and psychological well-being. This study followed the regulatory ethical requirements of the IRB.
CHAPTER 4: RESULTS

Overview

This study was designed to explore how cognitive (number sense), emotional, and motivational (mindset and motivation) components contribute to variance in math achievement among students with and without learning disabilities (LD). In order to understand this group of variables, each of the three independent variables (number sense, mindset, and motivation) was measured for middle school students with LD and students without LD and the relationships with math achievement were investigated. Additionally, this study sought to understand middle schoolers’ perceptions of motivation and mindset factors on their math achievement, as well as explore other potential factors related to their positive and/or negative experiences in math. To understand their perceptions of their personal experiences in depth, their stories, through interviews, were explored as the primary medium for data collection. In order to answer these questions, a mixed method was used to explore variables related to math achievement. An analysis of both qualitative and quantitative data was conducted separately and then integrated to synthesize the findings to enhance understandings of the variables associated with math achievement. The analysis of data collected was examined to answer research questions:

Research Questions.

1. Do number sense, math motivation, and mindset predict the dependent variable of math achievement in middle school students?
   a. After controlling for disability status, are there differences in math achievement based on number sense, mindset, and motivation?
   b. After controlling for number sense, are there differences in math achievement...
based on mindset and motivation?

2. Do teacher evaluations of student number sense correlate to scores on the IA numeracy scale (Gittens, 2015)?

3. How do middle school students with and without disabilities perceive mindset and motivation relate to their experiences with and achievement in math?

4. How do middle school students perceive positive and negative experiences with achievement in math?

Data Collection

The study sample included 6th, 7th, and 8th grade students in a public charter school in Indianapolis, Indiana. The sample was collected in May of 2018. A total of 60 students participated, 30 students in each group (students with LD and without LD). However, incomplete questionnaires and unavailability of past ISTEP scores for some students resulted in missing data. Out of the 60 students who participated, the data from 48 students were analyzed.

Analysis

Screening process: Descriptives. The descriptive statistics for all variables and correlations served to screen the data for assumptions of ANCOVA. The screening analysis was conducted in SPSS. The sample contained a total of 48 students. There were students with LD (Special Education) and without LD (General Education) for each grade level (see Table 6). There were 26 students without LD and 22 students with LD. The descriptive statistics of the variables, which includes means and standard deviations for the entire sample, were analyzed (see Table 7). Students with LD scored lower on measures of each variable than students without LD (see Table 8). In addition, Pearson’s
correlation coefficients for the variables (number sense, motivation, mindset, and math achievement) were calculated (see Table 8). As shown in the table, only the correlation between number sense and math achievement was statistically significant, $r = .476$. The other correlations were not statistically significant. Overall, students without LD scored higher on all measures (math achievement, number sense, mindset, and motivation) when compared to their peers with LD.

A priori power analysis for ANCOVA indicated that the sample size needed to detect a medium effect size ($f = 0.25$) was between 52 (power of 0.80) and 84 (power of .95). In the present study, there was limited sample size, which added challenges in detecting significance due to a lack of power. In addition, prior to conducting the ANCOVA, assumption testing was conducted for linearity, normality, sphericity, homogeneity of variance, and group independence (Warner, 2013).

Table 6

<table>
<thead>
<tr>
<th>GRADE</th>
<th>General Education</th>
<th>Special Education</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>13</td>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>22</td>
<td>48</td>
</tr>
</tbody>
</table>

*Note. General Education = students without LD, Special Education = students with LD. The highest participation was in 6th grade (N = 19).*
Table 7

*Group Statistics for each Variable (Number Sense, Motivation, Mindset, and Math Achievement)*

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISTEP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN</td>
<td>26</td>
<td>568.38</td>
<td>41.77</td>
</tr>
<tr>
<td>SPED</td>
<td>22</td>
<td>515.23</td>
<td>36.47</td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>544.02</td>
<td>47.32</td>
</tr>
<tr>
<td>Mindset</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN</td>
<td>26</td>
<td>3.00</td>
<td>.78</td>
</tr>
<tr>
<td>SPED</td>
<td>22</td>
<td>2.94</td>
<td>.74</td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>2.97</td>
<td>.75</td>
</tr>
<tr>
<td>Motivation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN</td>
<td>26</td>
<td>4.21</td>
<td>.52</td>
</tr>
<tr>
<td>SPED</td>
<td>22</td>
<td>3.76</td>
<td>.96</td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>4.00</td>
<td>.78</td>
</tr>
<tr>
<td>NS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN</td>
<td>26</td>
<td>78.69</td>
<td>7.66</td>
</tr>
<tr>
<td>SPED</td>
<td>22</td>
<td>71.77</td>
<td>6.55</td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>75.52</td>
<td>7.91</td>
</tr>
</tbody>
</table>

*Note.* NS = Number Sense, ISTEP = Math Achievement, GEN = students without LD, and SPED = students with LD.
Table 8

*Correlation between each Variable (Number Sense, Motivation, Mindset, and Math Achievement) for both groups (students without LD and student with LD) combined*

<table>
<thead>
<tr>
<th>Measure</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ISTEP</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Motivation</td>
<td>.197</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Mindset</td>
<td>.105</td>
<td>-.110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. NS</td>
<td>.476*</td>
<td>.223</td>
<td>-.067</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Only number sense and math achievement showed statistically significant correlation, NS = Number Sense, ISTEP = Math Achievement. **p < 0.01 (2-tailed).

**Analysis 1: Interrater Reliability.** An interrater reliability analysis was used to examine whether teachers’ evaluations of students’ number sense correlated to the IA numeracy scale (Gittens, 2015). Before administering the numeracy scale, the teachers were interviewed for their evaluations of students’ number sense. In addition to the author, two additional teachers (a math teacher and a special education teacher) were recruited to evaluate each student’s number sense. They rated the student’s number sense based on Gitten’s definition of numeracy, “critical thinking applied to a quantitative context” (Gittens, 2015, p. 2). They rated on a scale of one through five (1=low, 2=moderately low, 3=average, 4=moderately high, 5=high). The mean and standard deviation of the evaluation scores of three teachers were calculated (see Table 9) and the mean of the ratings from three teachers were correlated to each other (see Table 10). The mean of the teachers evaluations was also correlated to the scores on the IA numeracy
scale (Gittens, 2015) (see Table 11). The average measures of the intraclass correlation coefficient (ICC) for the teacher ratings and scores on the IA numeracy scale (Gittens, 2015) was .885 (higher than .7, which is a cutoff of significance for ICC) (95% CI = .814 - .932), F (47, 94) = 8.692, \( p < 0.001 \), indicating high interrater reliability (Hallgren, 2012). Thus, a high degree of interrater reliability was found among the three teacher evaluations. The Cohen’s kappa was .91, which indicated high agreement, in which 82%-100% of the data are reliable (Cohen, 1960). In addition, the scores of teacher evaluations were statistically significantly correlated with number sense scores on the IA numeracy scale (Gittens, 2015), \( r = .499, p < 0.001 \).

Table 9

*Statistics of Teacher Ratings on Student’s Number Sense*

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR1</td>
<td>3.73</td>
</tr>
<tr>
<td>TR2</td>
<td>4.13</td>
</tr>
<tr>
<td>TR3</td>
<td>3.69</td>
</tr>
</tbody>
</table>

*Note.* TR1 = teacher rating of Teacher 1, TR2 = teacher rating of Teacher 2, TR3 = teacher rating of Teacher 3
Table 10

*Correlations Between Teacher Ratings of Three Teachers*

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.TR1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.TR2</td>
<td></td>
<td>.621</td>
<td></td>
</tr>
<tr>
<td>3.TR3</td>
<td>.951</td>
<td>.592</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* TR1 = teacher rating of Teacher 1, TR2 = teacher rating of Teacher 2, TR3 = teacher rating of Teacher 3

Table 11

*Intraclass Correlation Coefficient*

<table>
<thead>
<tr>
<th></th>
<th>Intraclass Correlation&lt;sup&gt;b&lt;/sup&gt;</th>
<th>95% Confidence Interval</th>
<th>F Test with True Value 0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Upper Bound</td>
</tr>
<tr>
<td>Average Measures</td>
<td>.885&lt;sup&gt;c&lt;/sup&gt;</td>
<td>.814</td>
<td>.932</td>
</tr>
</tbody>
</table>

**Analysis 2: ANOVA.** In this study, one-way ANOVAs were conducted to determine whether there were statistically significant differences between the means of the groups (students with and without LD) on each variable (number sense, motivation, mindset, and math achievement). There were statistically significant differences between the two groups for math achievement and number sense (see Table 12). Students without LD scored statistically significantly higher in math achievement ($F(1,46) = 21.647, p < 0.001, \eta^2 = 3.201$), number sense ($F(1,46) = 11.077, p < 0.001, \eta^2 = 0.002$), and also scored higher on motivation, although not at a statistically significant level ($F(1,46) = 4.170, p = 0.047, \eta^2 = 0.083$). However, for motivation, the groups were unequal and
homogeneity of variance was violated. There were some violations in the normality of the data distribution due to different outliers and significance levels ($F = 7.240, p = 0.010$), but the one-way ANCOVA was still processed.

Two-way ANOVAs were conducted between groups (students without LD and with LD) and each variable (number sense, motivation, and mindset) in relation to math achievement as the outcome. This two-way ANOVA was conducted to examine the effect of each variable on math achievement between two groups (students with LD and students without LD). There were significant effects of disability status on math achievement for number sense and mindset, $F(1,32) = 12.673, p = < 0.001, \eta^2 = 0.28$ and $F(1,18) = 1.350, p = .004, \eta^2 = 0.60$, respectively (see Table 13 and 15). There was no significant effect of disability status on math achievement for motivation ($p = .305$, see Table 14). Students without LD overall scored higher than students with LD in math achievement (see Figure 2 and Figure 3).

Table 12

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
<th>$\eta^2$</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISTEP</td>
<td>1</td>
<td>33672.962</td>
<td>33672.962</td>
<td>21.647</td>
<td>.000</td>
<td>.3201</td>
<td>.995</td>
</tr>
<tr>
<td>Error</td>
<td>46</td>
<td>71556.017</td>
<td>1555.566</td>
<td></td>
<td></td>
<td>.083</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>47</td>
<td>105228.979</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motivation</td>
<td>1</td>
<td>2.387</td>
<td>2.387</td>
<td>4.170</td>
<td>.047</td>
<td>.083</td>
<td>.516</td>
</tr>
<tr>
<td>Error</td>
<td>46</td>
<td>26.329</td>
<td>.572</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>47</td>
<td>28.716</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Note
The groups are students without LD and students with LD. NS = number sense.

#### Table 13

*Two-Way ANOVA with Groups and Number Sense with Math Achievement as Dependent Variable*

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
<th>\eta^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>8369.992</td>
<td>8</td>
<td>1046.249</td>
<td>.709</td>
<td>.681</td>
<td>.151</td>
</tr>
<tr>
<td>Group</td>
<td>18689.489</td>
<td>1</td>
<td>18689.489</td>
<td>12.673</td>
<td>.001</td>
<td>.284</td>
</tr>
<tr>
<td>NS *</td>
<td>12483.158</td>
<td>6</td>
<td>2080.526</td>
<td>1.411</td>
<td>.241</td>
<td>.209</td>
</tr>
<tr>
<td>Group</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>47193.414</td>
<td>32</td>
<td>1474.794</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. Results of two-way ANOVA of the significant effect of disability group on math achievement. General Education represents students without LD and Special Education represents students with LD. NS = Number Sense, ISTEP = Math Achievement.

Table 14

Two-Way ANOVA with Groups and Motivation with Math Achievement as Dependent Variable

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p.</th>
<th>\eta^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>2090.770</td>
<td>1</td>
<td>2090.770</td>
<td>1.105</td>
<td>.305</td>
<td>.050</td>
</tr>
<tr>
<td>Motivation</td>
<td>28769.901</td>
<td>21</td>
<td>1369.995</td>
<td>.724</td>
<td>.767</td>
<td>.420</td>
</tr>
<tr>
<td>Group *</td>
<td>3575.978</td>
<td>4</td>
<td>893.995</td>
<td>.473</td>
<td>.755</td>
<td>.083</td>
</tr>
<tr>
<td>Motivation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>39726.117</td>
<td>21</td>
<td>1891.720</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Group = disability status.
Table 15

**Two-Way ANOVA with Groups and Mindset with Math Achievement as Dependent Variable**

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p.</th>
<th>η²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mindset</td>
<td>36782.204</td>
<td>20</td>
<td>1839.110</td>
<td>1.350</td>
<td>.263</td>
<td>.600</td>
</tr>
<tr>
<td>Group</td>
<td>14588.758</td>
<td>1</td>
<td>14588.758</td>
<td>10.710</td>
<td>.004</td>
<td>.373</td>
</tr>
<tr>
<td>Mindset *</td>
<td>7396.668</td>
<td>8</td>
<td>924.583</td>
<td>.679</td>
<td>.704</td>
<td>.232</td>
</tr>
<tr>
<td>Group</td>
<td>14588.758</td>
<td>1</td>
<td>14588.758</td>
<td>10.710</td>
<td>.004</td>
<td>.373</td>
</tr>
<tr>
<td>Error</td>
<td>24518.250</td>
<td>18</td>
<td>1362.125</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Group = disability status.

**Figure 3.** Results of two-way ANOVA of the significant effect of disability group on math achievement. General Education represents students without LD and Special Education represents students with LD. NS = Number Sense, ISTEP = Math Achievement.
Analysis 3: ANCOVA. ANCOVA was used to control for the effects of the
covariate and explain the effects of selected primary independent variables on math
achievement. Three one-way ANCOVA’s were conducted to compare the effect of each
outcome variable (disability group, mindset, and motivation) while controlling for
number sense (see Tables 16, 17, and 18). The main effects of disability group, mindset,
and motivation were assessed after the math achievement scores were adjusted for
differences accounted for by number sense. Number sense was a source of variation not
controlled for in the design of the experiment, but believed to affect math achievement.
The group means on math achievement were adjusted so that they represented groups
with the same means on the number sense. The use of number sense as a covariate in the
analysis served to reduce the unexplained variation. Thus, the purpose of ANCOVA was
to determine whether the mean differences among the variables on adjusted math
achievement were likely to have occurred by chance. Prior to conducting the ANCOVA,
assumption testing was conducted for linearity, normality, sphericity, homogeneity of
variance, and group independence (Warner, 2013). Using correlation and ANOVA,
predictors were screened for correlation with the dependent variable. Number sense was
statistically significantly correlated with math achievement. Thus, ANCOVA analyses
were conducted with number sense as a covariate.

Statistical steps were dedicated to determine violations of assumptions such as
linearity, normality, homogeneity of variance, and group independence. A linear
relationship between dependent variable and any covariates is assumed for ANCOVA
(Huitema, 2011). The assumption of linearity was tested by comparing the variance
explained by fitted model to the unfitted model with the covariate raised to the higher
power. In the present study, the addition of number sense did not lead to a significant increase in the variation explained, $F(1,8) = .557, p = .786$. The normalcy of the distribution and sphericity were reviewed. ANOVA models assume that the errors of fitted model are distributed normally (Huitema, 2011). In the present study, the normal probability plots did not suggest violation of normality for math achievement ($W = .978, p = .876$) and mindset ($W = .968, p = .202$). However, the normality test suggested a possible violation of normality for number sense ($W = .947, p = .031$) and motivation ($W = .95, p = .001$). The violation for number was relatively mild ($p = .031$) thus the violation was probably inconsequential (Huitema, 2011). The homogeneity of variance was screened, all met assumptions for normality for ANCOVA ($F = .124, p = .796; F = 1.298, p = .261$). However, the homogeneity of variance was not met for the ANCOVA with number sense as a covariate and motivation as the independent variable ($F = 2.636, p = .010$), so degrees of freedom were adjusted. Adjusting degrees of freedom overcame the violation by increasing the $p$-value above the critical significance of 0.05 (Pandey & Bright, 2008) In addition, Mauchly’s Test of Sphericity indicated that the assumption of sphericity had been violated, $\chi^2(2) = 118.915$, d.f. = 2, $p < .001$. While there was violation of sphericity, the violation was not impactful for ANCOVA (Huitema, 2011). In addition, ANCOVA assumes that the levels of independent variables are specified in study design (Huitema, 2011). In the present study, number sense, motivation, and mindset were specified during study design. Thus, it met the assumption for group independence. While some of the assumptions were not met, the violations were inconsequential.
The results of ANCOVA indicated that there was a statistically significant difference in math achievement between the groups (students with LD and students without LD) when their means were adjusted for number sense, \( F(1,45)= 11.483, p < 0.001 \) (see Table 16). Furthermore, with number sense as a covariate, there was no change in the effects of the other variables on math achievement; each remained statistically insignificant (see Table 17 and 18). There were no statistically significant differences between different levels of motivation or mindset between groups when controlling for number sense. However, there were statistically significant differences in math achievement for the group with LD after controlling for number sense and, \( F(1,45) = 11.483, p < 0.001, d = .203 \). This finding suggests that the effect of disability status remains statistically significant even after controlling for the covariate.

Table 16

*ANCOVA with Number Sense as Covariate with Disability as Independent Variable and Math Achievement as Dependent Variable*

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
<th>( \eta^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>6716.388</td>
<td>1</td>
<td>6716.388</td>
<td>4.661</td>
<td>.036</td>
<td>.094</td>
</tr>
<tr>
<td>Group</td>
<td>16546.185</td>
<td>1</td>
<td>16546.185</td>
<td>11.483</td>
<td>.001</td>
<td>.203</td>
</tr>
<tr>
<td>Error</td>
<td>64839.630</td>
<td>45</td>
<td>1440.881</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. NS = Number Sense, Group = Disability Status.*
Table 17

**ANCOVA with Number Sense as Covariate with Mindset as Independent Variable and Math Achievement as Dependent Variable**

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
<th>η²</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>8055.882</td>
<td>1</td>
<td>8055.882</td>
<td>5.172</td>
<td>.031</td>
<td>.166</td>
</tr>
<tr>
<td>Mindset</td>
<td>40887.914</td>
<td>20</td>
<td>2044.396</td>
<td>1.313</td>
<td>.254</td>
<td>.502</td>
</tr>
<tr>
<td>Error</td>
<td>40497.901</td>
<td>26</td>
<td>1557.612</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* NS = Number Sense.

Table 18

**ANCOVA with Number Sense as Covariate with Motivation as Independent Variable and Math Achievement as Dependent Variable**

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
<th>η²</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>3550.199</td>
<td>1</td>
<td>3550.199</td>
<td>1.944</td>
<td>.176</td>
<td>.072</td>
</tr>
<tr>
<td>Motivation</td>
<td>35729.290</td>
<td>21</td>
<td>1701.395</td>
<td>.932</td>
<td>.562</td>
<td>.439</td>
</tr>
<tr>
<td>Error</td>
<td>45656.525</td>
<td>25</td>
<td>1826.261</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* NS = Number Sense.

Another ACNOVA analysis was conducted with disability status as a covariate. The relationships among number sense, motivation, and mindset with math achievement were explored, after controlling for disability status. The disability status was assigned as a covariate to explore the effects of other variables in relation to math achievement while controlling the effect of LD (see Tables 19, 20, and 21). The use of disability status as a covariate in the analysis served to reduce the unexplained variation. Statistical steps were dedicated to determine violations of assumptions such as linearity, normality, homogeneity of variance, and group independence (Warner, 2013). The homogeneity of
variance and other assumptions such as normality were screened, all met assumptions for normality for ANCOVA, \(F = .678, p = .707; F = 1.914, p = .509; F = .396, p = .207\) except for motivation \(F = 2.636, p = .010\). Mauchly’s Test of Sphericity indicated that the assumption of sphericity had been violated, \(\chi^2(2) = 118.915, \text{d.f.} = 2, p < .001\).

Although some assumptions were not met, the violations were inconsequential for the ANCOVA analysis. The results of ANCOVA indicated that after controlling for disability status, none of the variables were statistically significantly related to math achievement, indicating no statistically significant effect of number sense, motivation, or mindset on math achievement.

Table 19

**ANCOVA with Disability as Covariate with Number Sense as Independent Variable and Math Achievement as Dependent Variable**

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
<th>(\eta^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>14312.665</td>
<td>1</td>
<td>14312.665</td>
<td>9.114</td>
<td>.005</td>
<td>.193</td>
</tr>
<tr>
<td>NS</td>
<td>11879.445</td>
<td>8</td>
<td>1484.931</td>
<td>.946</td>
<td>.492</td>
<td>.166</td>
</tr>
<tr>
<td>Error</td>
<td>59676.573</td>
<td>38</td>
<td>1570.436</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. Group = disability status.*

Table 20

**ANCOVA with Disability as Covariate with Motivation as Independent Variable and Math Achievement as Dependent Variable**

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
<th>(\eta^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>5904.629</td>
<td>1</td>
<td>5904.629</td>
<td>3.409</td>
<td>.077</td>
<td>.120</td>
</tr>
<tr>
<td>Motivation</td>
<td>28253.922</td>
<td>21</td>
<td>1345.425</td>
<td>.777</td>
<td>.720</td>
<td>.395</td>
</tr>
<tr>
<td>Error</td>
<td>43302.095</td>
<td>25</td>
<td>1732.084</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. Group = disability status.*
Table 21

ANCOVA with Disability as Covariate with Mindset as Independent Variable and Math Achievement as Dependent Variable

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
<th>$\eta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>16638.866</td>
<td>1</td>
<td>16638.866</td>
<td>13.555</td>
<td>.001</td>
<td>.343</td>
</tr>
<tr>
<td>Mindset</td>
<td>39641.100</td>
<td>20</td>
<td>1982.055</td>
<td>1.615</td>
<td>.125</td>
<td>.554</td>
</tr>
<tr>
<td>Error</td>
<td>31914.918</td>
<td>26</td>
<td>1227.497</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14311245.000</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Group = disability status.

**Analysis 4: Qualitative analysis.** To understand the participants’ perceptions of motivation and mindset factors on their math achievement, I conducted 15 interviews which I recorded then transcribed for the analysis. I used analytic memos to identify and refine important concepts. In the process of analysis, I compared the perceptions reported by participants by the disability groups. The coding process allowed me to explore their perceptions about their positive and negative experiences in math. The participants reported positive and/or negative experiences which revealed that students tend to desire to do more math when they are successful, and unsuccessful experiences can lead to frustration and boredom. In addition, students with LD tend to report more negative motivation compared to students without LD.

**Coding.** After transcribing the interviews, I read the interviews to determine which information was relevant to the study and most responsive to the research questions in preparation for the analysis. A grounded theory approach was used that was based on priori codes of the variables. Based on the grounded theory approach, I analyzed the interviews through open coding. By coding early-in-depth interview data, I could identify implicit and explicit statements and keep focused through data analysis.
(Charmaz, 2006). I analyzed through two cycles of coding. The first cycle of coding was open coding. While reading the transcripts, I used jottings to document the initial analytic thoughts, to uncover the meaning, ideas, and thoughts embedded in the text. I labeled concepts and developed categories based on their properties. The concepts were based on the words students used to describe their perceptions. The second cycle of focused coding filtered and highlighted the data to generate themes and concepts of the study (Saldana, 2013). Multiple codes that shared a large, common idea were color coded and grouped into content-based themes.

In this phase of coding, the codes assigned to each response were words that described thoughts, actions, or feelings. The purpose of the second cycle of coding was to select significant codes from the initial codes to synthesize the data. It allowed me to advance the theoretical direction of the study shaping the concept that was focused a priori. I used QDA Miner software to analyze the data. The coding led to several themes of students’ experiences with motivation, mindset and math achievement. This process of collapsing codes clarified concepts and ideas that produced themes that provide abstract understanding of the responses. Theoretical sorting of the themes helped the process of generating conceptual understanding. The generated analysis was grounded in the data to understand the perceptions of the participants. The findings that follow describe the synthesis of the primary themes that emerge from the responses. The students who participated in the interview are labeled with numbers and the demographic information is provided (see Table 22).
Table 22

Demographic and Disability Status for Students who Participated in the Interview

<table>
<thead>
<tr>
<th>Student #</th>
<th>Disability Status</th>
<th>Grade</th>
<th>Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>G</td>
<td>7</td>
<td>M</td>
</tr>
<tr>
<td>2</td>
<td>G</td>
<td>8</td>
<td>F</td>
</tr>
<tr>
<td>3</td>
<td>G</td>
<td>6</td>
<td>M</td>
</tr>
<tr>
<td>4</td>
<td>G</td>
<td>7</td>
<td>M</td>
</tr>
<tr>
<td>5</td>
<td>G</td>
<td>6</td>
<td>F</td>
</tr>
<tr>
<td>6</td>
<td>G</td>
<td>7</td>
<td>F</td>
</tr>
<tr>
<td>7</td>
<td>G</td>
<td>8</td>
<td>M</td>
</tr>
<tr>
<td>8</td>
<td>S</td>
<td>7</td>
<td>S</td>
</tr>
<tr>
<td>9</td>
<td>S</td>
<td>6</td>
<td>M</td>
</tr>
<tr>
<td>10</td>
<td>S</td>
<td>7</td>
<td>M</td>
</tr>
<tr>
<td>11</td>
<td>S</td>
<td>6</td>
<td>M</td>
</tr>
<tr>
<td>12</td>
<td>S</td>
<td>7</td>
<td>M</td>
</tr>
<tr>
<td>13</td>
<td>S</td>
<td>7</td>
<td>F</td>
</tr>
<tr>
<td>14</td>
<td>S</td>
<td>6</td>
<td>F</td>
</tr>
<tr>
<td>15</td>
<td>S</td>
<td>8</td>
<td>M</td>
</tr>
</tbody>
</table>

Note. G = Students without LD, S = Students with LD, F = female, M = male

Motivation. For the motivation scale, the students without LD overall reported that they “enjoyed” math because they were learning new things and it was easy for them. Most of them reported that they felt “confident” because when they worked hard, they got high scores on their assessments. Some students without LD (Students 3, 4, and 5)
said that they also “apply what they learn in their lives” when they learn new things. The
students recited their experiences of calculating money and time outside of school and
understanding numbers in their daily lives. In addition, they were “excited to learn math.”
For some of them (Students 3, 4, and 6), it was their favorite subject. For some (Students
4 and 7), they considered math as a “requirement for school”. Overall, students without
LD shared positive attitudes about math based on their perceptions of motivation. They
felt confident in math, and expressed their enjoyment and excitement in math.

On the other hand, students with LD reported lower motivation compared to
students without LD. Some of them (Students 9, 14, and 17) said, “if I don't understand, I
get frustrated and bored.” Others (Students 8, 10, and 12) shared that math is “difficult
with multi-steps but easier with step-by-step examples.” Students with LD, overall,
shared low confidence in math. Student 15 said, “From 1 through 5, I am 2 in my
confidence in math.” Some claimed, “I don’t understand so I have to practice more.”
They also described their difficulty with multi-step problems in math, “Math is easy for
the first or second step. The other steps get harder.” Students with LD, overall, expressed
their frustration and boredom when they struggle and their low confidence due to
difficulties in math.

*Math is easy.* Students without LD shared their thoughts about what is easy about
math. Most of the students specified certain “math topics” that were easy for them, which
included fractions, decimals, and negatives. Students without LD also explained their
stories about their confidence in completing the most difficult homework problems in
math. They referred about their past challenges of not understanding some topics and
independently working on them at home without any help. For some students (Students 3,
4, and 5), every homework assignment was easy for them. Most of the students without LD (Students 1, 3, 4, and 6) claimed that they felt confident because when they “focus, they learn something new”. Students without LD perceived math as easy for them and felt confident to work independently.

In contrast, students with LD (Students 10 and 15) mentioned that math can be “hard until they understand it with practice.” Students 8 and 11 indicated “the use of calculator” makes math easy for them. For some students (Students 9, 14, and 15), they were not confident because “math is hard and they are not much good at it and it is boring.” Students with LD also recounted that math is hard for them when they are trying to “understand the problem,” and when they work on math on their own without knowing what to do. Student 8 talked about writing down the numbers and she did not know what the number was. For some students with LD (Students 12, 13, and 15), “problems with multiple steps” can be hard. They also specified topics that were challenging, which include slopes, equations, fractions, and equation. Students with LD perceived math as hard for them until they finally understand.

Working hard. For some students without LD (Students 3, 4, and 5), they enjoyed math because it helps them prepare for what they need for the “future and gives them good challenge.” Some described the times when they worked hard and practiced examples that they understood quickly. A few (Students 1, 3, and 5) recited their experiences of “learning a new topic” on the first day, which was hard until they practiced. Students without LD overall expressed their motivation toward learning math because they learn new topics and gives them challenge.
In addition, students with LD also expressed their excitement about learning math because it is important for them to learn new topics and it can be easier after they practice and understand it. Some students with LD (Students 10 and 13) mentioned when “teachers explain and provide examples”, it makes math easier for them. However, they also said that math is hard for them when the teacher does not explain in details and they don’t understand the questions. After the teachers “explain with examples” they enjoy math. Learning new things about math can be fun but it is hard when they do not get the full explanation of the topic and do not comprehend. This can cause them to get “frustrated and bored.” Some students with LD (Students 11 and 14) referenced their own experiences of not being able to work on difficult homework problems on some topics and getting frustrated and bored which led them to stop doing their homework.

**Mindset.** To explore students’ perceptions of mindset on math achievement, students answered questions on their own interpretation of intelligence and their experiences in math classes. Both groups (students with LD and without LD) shared similar stories. Overall, the students answered that they have growth mindsets, in which they believed that basic abilities or intelligence can be developed through effort and practice.

**Intelligence.** Students were asked to describe intelligence in their own words. Students without LD interpreted intelligence as “how much you know” and being “above average.” Most of them described intelligence as “grades in school” and “the ability to understand quickly.” Others had less traditional notions about intelligence. For example, Student 4 said, “I don’t think it is just about grades. I think when somebody is smart, it is the decisions that they make and how they apply.” Some students mentioned how it
comes from “experience and how you are raised.” Student 3 said, “It comes from experience because it is hard to be born super smart.” Student 2 mentioned that “intelligence can increase and decrease.” As an example, she stated, “if you do not practice and if you are not dedicated, your grades can go down.” Overall, students without LD who participated in the interview, believed that intelligence comes from practice, effort, and staying attentive in class. Students without LD also shared their experiences of when they felt intelligent. Most of them shared their stories about getting “good grades and understanding certain topics”. For example, Student 4 said, “I felt intelligent when I got good grades in math and I understood everything in my math class.”

Students with LD also replied similarly as students without LD did, that “intelligence comes from experience.” Some students mentioned the “teacher” or “help” that they received helped them to become smart. They named some teachers who helped them get “smarter” in their math classes. Some students expressed their beliefs that intelligence comes “from their parents” and you are “born with the intelligence.” Student 14 expressed intelligence as for people who are intelligent, “things are easy for them.” Students with LD also shared their experiences of when they were feeling intelligent. They shared their experiences of getting good grades on quizzes and tests, and they felt very intelligent in math. Some students with LD felt achieved and accomplished. Others expressed their “excitement” when practicing and studying resulted in positive outcomes.

Ability to change intelligence. Emergent themes overlapped between students with LD and students without LD. Students’ mindsets can be described by their beliefs about whether intelligence cannot be changed or can be developed through repeated practice. In response to the interview question regarding their beliefs in their ability to
change their intelligence, some students with LD and without LD reported that you can change your intelligence, like “your personality.” Most of them said that you can change your ability by “trying your best by practicing.” Students also referred to their “grades” and how they got “help” and did better on their assessments. They shared how their parents and teachers helped them to get better grades and felt more intelligent. Others had less traditional notions about their ability to change intelligence by responding, “You have to act normal” (Student 10) and “You can act like you are not smart” (Student 15). These responses both included the word “act” in their response to intelligence. This can suggest these students focus on comparisons to others and also other students’ perceptions or their portrayal to others.

Overall, the students without LD and with LD responded that they have growth mindsets. They had perceptions that you can “change how smart you are with practice.” Student 5 (without LD) stated, “At the beginning of the year, I did not understand math. Now, at the end of the year, I got straight A’s.” Another student without LD (Student 6) shared, “If you start slacking and spend more time with your friends after school and not do your homework, that can bring you down. If you are not studying and pay more attention, or slack off, your grades would slack off. You can change how intelligent you are.” A student with LD (Student 9) also stated, “I feel like it depends on the person you are. Like your personality and stuff that can change.” Student 15 (with LD) shared his story, “I have more growth mindset. Math was Ds and Fs. My grades changed to A’s and B’s. What you learn can change what you know.” Student 14 expressed, “I was thinking you can change how smart you are because if you learn more things then you can increase intelligence by learning new things.” Other students with LD responded that
they have growth mindsets because they believed that they can get better grades and get smarter when they try.

**Analysis 5: Integrated convergent analysis.** An integrated analysis can provide meaningful insights into relationships between number sense, motivation, mindset, math achievement, and disability. A convergent design with joint display can represent results with a theme-by-statistics display, the qualitative and quantitative results (Merriam et al., 2002). In this study, a convergent integrated analysis of levels of math achievement and motivation and mindset revealed that students tend to desire to do more math when they are successful, and unsuccessful experiences can lead to frustration and boredom. In addition, students with LD tend to report more negative motivation compared to students without LD.

**Math achievement and motivation.** Students with high, medium, or low math achievement, in general, shared similar experiences related to mindset. However, there were small nuanced differences in their responses in their perceptions of motivation in math. Even though the statistical analysis of quantitative data on motivation and math achievement ($r = .197$) was not significant, the synthesis of the quantitative and qualitative data suggested that in general, compared to students with high math achievement, students with low math achievement reported more negative experiences in math motivation toward their experiences with and achievement in math. For example, students with high math achievement mentioned, “I enjoy math from a day to day basis. I would do it everyday and I am dedicated” and “math is easy to me. I want to learn new things every day.” One student with high math achievement (Student 2) answered, “I am 100% confident that I am going to get a good grade because I practice math at home and
on the phone. I use math every single part of the day.” Another student (Student 5) also showed high motivation, “I am excited to learn math because math is fun and it can be educated at the same time.”

In contrast, students with low math achievement reported, “Math is hard a little but until you understand it with practice. After you understand, you enjoy it”. Student 15 described their experience in math, “From 1 through 5, I am 2 in my confidence in math. For some, I don’t understand so I have to practice more.” Student 15 with low math achievement stated, “Math is easy for the first or second step. The other steps get harder. Math is also hard when you are doing it on your own if you don’t know what you are doing.” Based on the students’ answers on motivation between students with high math achievement and low math achievement, math achievement and motivation are correlated in the synthesis of the integrated analysis of the quantitative and qualitative data.

Number sense and motivation. This study used an integrated analysis to examine how students with low, medium, and high scores on a measure of number sense perceived their experiences in math. The statistical analysis of the correlations between number sense and motivation was not significant, $r = .223$. There are emergent themes of motivation for high, medium, and low number sense. Students with LD with low number sense reported “I never enjoy doing math,” and “I am not confident in math because math is boring.” In contrast, some students with high number sense shared positive motivation, “I am very excited, 10 out of 10.” One student with high number sense (Student 2) answered, “It depends on what we are doing in math, but I feel confident over all. Some of the stuff can be hard.” The qualitative data demonstrate nuanced differences in their answers on motivation between students with high number sense and low number sense.
Thus, the synthesis of the integrated analysis of the quantitative and qualitative supports a significant correlation between number sense and motivation.

**Differences between students with LD and without LD.** To explore students’ perceptions of mindset on math achievement, students answered questions about their own interpretation of intelligence and their experiences in math classes. There were emergent themes of “intelligence comes from experience” and “change how smart you are with practice” for both groups. Student 1 responded, “It depends on your own self, if you wanna be smart or if you wanna work, you can push yourself to learn more. Some people try to learn and get smart but some people don’t try.” The correlation between mindset and math achievement was not statistically significant ($r = .105$). This aligns with the students’ experiences with math based on their perceptions of mindset in math. Students with LD described that “intelligence can come from getting help.” Similarly, students without LD described intelligence as “intelligence comes from practice, effort, and dedication” and "it is hard to be born smart.” The statistical analysis of the quantitative results reflects the students’ reported perceptions of the influences of mindset in math. They, in general, believed that they could change how smart they are with dedication and hard work.

Conversely, students with LD indicated more negative experiences in math based on their perceptions of motivation in math, as shown in the quantitative results. There were significant differences between the groups for motivation, $F(1,46) = 4.170, p = 0.047$. The responses from the interview also supported the statistical result, in which there were nuanced differences in their answers between students with LD and without LD. For instance, Student 4 (student without LD) expressed positive experiences in math,
“I am very confident because I know if I put my mind to something, I can do it.” In contrast, Student 14 (student with LD) said, “I enjoy doing math on 5/10 because sometimes learning new things about math is fun and sometimes when I do hard math and it gets frustrating and I get bored. Sometimes, I am not listening and I don’t get the full explanation of the topic and I don’t understand and I get frustrated.” Students without LD acknowledged that math can be hard but also can be easy with practice and effort, whereas, students with LD mentioned how when they are unsuccessful, they get frustrated and bored. Therefore, through the integrated analysis, compared to students without LD, students with LD reported more negative experiences in math motivation toward their experiences with and achievement in math (see Tables 23, 24, and 25).
Table 23

A Joint Display of Descriptive Statistics of the Variables and Main Themes of Motivation and Mindset from the Interview

<table>
<thead>
<tr>
<th></th>
<th>Motivation</th>
<th>Mindset</th>
<th>Experiences with motivation</th>
<th>Experiences with mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>with LD</td>
<td>N=22</td>
<td>N=22</td>
<td>● “if don't understand, get frustrated bored”</td>
<td>● “Intelligent means knowing a lot of things.”</td>
</tr>
<tr>
<td></td>
<td>minimum=1.4</td>
<td>minimum=1.3</td>
<td>● “like learning new things in math”</td>
<td>● “intelligence is getting grades on work, &quot;it is easy for them&quot;</td>
</tr>
<tr>
<td></td>
<td>maximum=5</td>
<td>maximum=4.4</td>
<td>● “difficult with multi-steps but easier with step-by-step guidance”</td>
<td>● “intelligence can come from getting help, practice, and can be born with intelligence”</td>
</tr>
</tbody>
</table>
|          | M =3.76    | M = 2.94 | ● “hard first but easier with practice” | ● "You would not be able to change it, you are stick with what you have"
|          | SD =0.96   | SD = 0.74 | ● “need math for future career” | ● "you can act smart or normal"
| without LD | N=26       | N=26     | ● “feel confident” | ● “intelligence is about how much you know” |
|          | minimum=3  | minimum=1 | ● “like learning new things” in | ● “intelligence is being above average” |

117
maximum = 5  maximum = 4.1  ● “easy with specific examples given”  ● “intelligence comes from practice, effort, and dedication”, "it is hard to be born smart"

M = 4.21  M = 3  ● “hard first but easier with practice”  ● “felt intelligent when received good grades and/or understood the topic”

SD = 0.52  SD = 0.78  ● “need math for future career”  ● "it depends on how you want to get smarter or not"

**Note.** ISTEP = Math Achievement

Table 24

*A Joint Display with Different Levels of Number Sense and Their Perceptions of Motivation and Mindset related to Math Achievement*

<table>
<thead>
<tr>
<th>Quantitative Data</th>
<th>Motivation</th>
<th>Mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS= 85</td>
<td>● “I enjoy math, from 1-10, it would be 8.”</td>
<td>● “Intelligence means the person is on the A game and know how to do everything.”</td>
</tr>
<tr>
<td>MO=3.6</td>
<td>● “It depends on what we are doing in math, but I feel confident over all. Some of the stuff can be hard.”</td>
<td>A game and know how to do everything.”</td>
</tr>
<tr>
<td>MS=3</td>
<td></td>
<td>“Intelligence comes from working hard and paying attention.”</td>
</tr>
<tr>
<td>ISTEP=587</td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Education</td>
<td>● “I am very excited, 10 out of 10.”</td>
<td></td>
</tr>
</tbody>
</table>
“When I am working, sometimes I feel I need help because without teachers, I don’t know what to do.”

“I felt very smart when I was getting all the answers right.”

“You can change your intelligence. It can increase or decrease. You can get smarter and learn more and you can also forget about everything.”

“I believe that I can still do better at where I am now.”

<table>
<thead>
<tr>
<th>Quantitative Data</th>
<th>Motivation</th>
<th>Mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS=78</td>
<td>“I like doing math a lot because in my future, that is the main thing that I need besides reading that I need to pursue my career.”</td>
<td>“Intelligence comes from the experience because if you pay attention, it would be easy. If you are used to getting good grades, you will keep getting good grades. You can change your grades, your grades does not determine how smart you are. Say you got an F, and you are the smartest kid in school, you can”</td>
</tr>
<tr>
<td>MO=4.7</td>
<td>“I am very confident in math because I know if I put my mind to something, I can do it. I am more confident because I want to learn new things so I can be successful.”</td>
<td></td>
</tr>
<tr>
<td>MS=3.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ISTEP=495</td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Education</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
● “For certain questions, when the teacher explains to you, it is easy for me to catch up because there are multiple steps and you can use what you are taught and apply to solve questions that you are given.”

● “I am very sure that I can do the most difficult homework problems in math because I know if I try, I can accomplish and get at least 85% of them right. If I am focused, I can get 100%.”

● “I felt accomplished and smart when I got the top academic math scholar. I wanted to improve and get more questions right.”

● “I think you can change how intelligent you are by how much effort you put into. It does not matter how smart a student is. You can still better yourself so you can learn new things.”

● “I am better now because I pay attention in class.”

● “Someone who is smart is in your eyes; they know a lot more than somethings that you never knew.”

● “Intelligence comes from experience. I got a 20% on one of the quizzes and I bring the F up if you put your time and stay dedicated and get an A.”

NS= 71
MO=3.2
MS=3.5
Special Education

● “I enjoy math a lot because it is one of my favorite subjects because it gives me good challenge.”

● “I am very confident in math because out of everything, math, I can do best.”
• “I am not that excited to learn math because it is not fun learning but I need to learn because it is essential when I get a job.”

• “Math is easy for me because there are different ways you can solve certain questions.”

• “You can change how smart you are and it depends on the person you are. Like your personality, you can change intelligence.”

• “I never enjoy doing math. I hate it because the substitute said I am not allowed to use calculator and I am allowed to use calculator.”

• “I am not confident in math because math is boring.”

• “Reading the questions is easy in math. Writing down the numbers is hard. I don’t went back and fixed it and did a better job. I wanted to solve whole bunch of questions. I wanted to make myself smarter.”

• “Intelligence means you are doing the work good”.

• “You are born with intelligent. You are born that way”.

• “I did well on my homework and felt happy because I did the right thing to learn math”.

• “You cannot change how intelligent you

<table>
<thead>
<tr>
<th>Quantitative Data</th>
<th>Motivation</th>
<th>Mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS= 67</td>
<td>“I never enjoy doing math. I hate it because the substitute said I am not allowed to use calculator and I am allowed to use calculator.”</td>
<td>“Intelligence means you are doing the work good”</td>
</tr>
<tr>
<td>MO=3.2</td>
<td></td>
<td>“You are born with intelligent. You are born that way”</td>
</tr>
<tr>
<td>MS=2.1</td>
<td></td>
<td>“I did well on my homework and felt happy because I did the right thing to learn math”</td>
</tr>
<tr>
<td>ISTEP=413</td>
<td></td>
<td>“You cannot change how intelligent you</td>
</tr>
</tbody>
</table>

Special Education
know what the number is.”

- “Math is hard with numbers because some of them are hard to add up.”
- “I break things some times. You can’t change how smart you are even if you try.”

Note. ISTEP = Math Achievement, NS = Number Sense, MO = Motivation, MS = Mindset

Table 25

*A Joint Display with Different Levels of Math Achievement and Their Perceptions of Motivation and Mindset related to Math Achievement*

<table>
<thead>
<tr>
<th>Quantitative Data</th>
<th>Motivation</th>
<th>Mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISTEP=604</td>
<td>• “I enjoy math from day to day basis. I would do it everyday basis and I am dedicated. Math is easy to me. I want to learn new things every day.”</td>
<td>• “To me, someone who is smart is bright, educational, and does not take long time to know something.”</td>
</tr>
<tr>
<td>NS= 64</td>
<td>• “I am 100% confident that I am going to get a good grade because I practice math at home and on the phone. I use math every single part of the day.”</td>
<td>• “Intelligence comes from your roots of your family. When my mom was in high school, she was in high ability. All of the ladies in my family is smart. It also depends on how hard you try.”</td>
</tr>
<tr>
<td>MO=4.7</td>
<td>• “I am 100% excited to learn math because”</td>
<td>• “I felt intelligent when I understood a</td>
</tr>
<tr>
<td>MS=3.4</td>
<td>• “I felt intelligent when I understood a”</td>
<td></td>
</tr>
<tr>
<td>General Education</td>
<td>• “I felt intelligent when I understood a”</td>
<td></td>
</tr>
</tbody>
</table>

122
“Math is fun and it can be educated at the same time.”

- “Math is easy because if you get the concept, it is going to be in your head for the rest of your life.”
- “Math can be hard when you enter a topic and it is the first day, until you practice.”
- “Every time I do my homework, I get 100%.”

Medium in Math Achievement

<table>
<thead>
<tr>
<th>Quantitative Data</th>
<th>Motivation</th>
<th>Mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISTEP=536</td>
<td>• “I enjoy math a lot but it can be hard”.</td>
<td>• “A smart person picks things up fast and they care about what they are learning about and try their best on every assignment no matter how hard it is.”</td>
</tr>
<tr>
<td>NS=71</td>
<td>• “I am not that confident but I end up getting a good grade.”</td>
<td></td>
</tr>
<tr>
<td>MO=4.3</td>
<td>• “I am very excited to learn math because math is my favorite subject because it is the easiest to me. Sometimes I struggle but not a lot.”</td>
<td>• “Intelligence comes from experience. You have to start from the easiest and work your way up to the hardest. You</td>
</tr>
<tr>
<td>MS=2.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
“Math is easy for me because I pick up things very fast. It can be hard sometimes because I don’t understand the way the question is phrased and I get the question wrong.”

“I felt intelligent in math when I studied for a quiz and got 100%. I felt happy and wanted to go home and show my mom.”

“If you are really smart and get A’s and B’s but if you start slacking and spend more time with your friends after school and not do your homework, that can bring you down. If you are not studying and not pay attention, or slack off, your grades would slack off.”

Low in Math Achievement

<table>
<thead>
<tr>
<th>Quantitative Data</th>
<th>Motivation</th>
<th>Mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISTEP=458</td>
<td>“Math is hard a little but until you understand it with practice. After you understand, you enjoy”</td>
<td>“Someone who is smart knows a lot more than others.”</td>
</tr>
<tr>
<td>NS=75</td>
<td></td>
<td>“Intelligence comes from studying hard and what parents taught them. It can be</td>
</tr>
<tr>
<td>MO=3.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MS=2.5</td>
<td>“When I come to math class, I have to be prepared for whatever comes for you.”</td>
<td></td>
</tr>
</tbody>
</table>
Special Education

focused because I am learning something new."

● “From 1 through 5, I am 2 in my confidence in math. For some, I don’t understand so I have to practice more.”

● “Math is easy for the first or second step. The other steps get harder. Math is also hard when you are doing it on your own if you don’t know what you are doing.”

● “When I got most of the questions right, I felt very intelligent. I got done before other students. I felt great and I wanted to do more.”

● “You can change how smart you are. You can practice more. You can act like you are not smart.”

Note. ISTEP = Math Achievement, NS = Number Sense, MO = Motivation, MS = Mindset
CHAPTER 5: DISCUSSION

The present study used an integrated analysis to explore how cognitive (number sense), emotional, and motivational (mindset and motivation) components contribute to variance in math achievement among students with and without learning disabilities (LD). The middle schooler’s perceptions of motivation and mindset factors on their math achievement and the predictors explained potential factors related to their positive and/or negative experiences with math. After conducting the study, the initial conceptual framework was revised. In the revised framework based on the present findings, number sense was statistically significantly correlated to math achievement, and additional integrated analysis indicated that there were differences in motivation between the two groups (students with LD and students without LD) and that motivation was correlated to number sense and math achievement. The revised framework can offer important insight to the field of special education, particularly regarding the predictors of math achievement for middle school students with LD. Specifically, the results may inform future research on the most effective way to structure interventions to support math achievement, future conceptual frameworks, and new perspectives on practice in education.

As hypothesized, the differences between students with LD and students without LD in math achievement were statistically significant. Further, the mean difference of the scores on number sense between the two groups (students with LD and without LD) was significant, which suggests the need to further investigate and understand the construct of number sense during the critical transitional period before high school. This study also found a statistically significant effect of number sense on math achievement. However,
there were no statistically significant effects of mindset or motivation on math
achievement. In addition, as anticipated, experiences shared by the students in this study
provided further evidence of the nuances of the predictors in relation to math
achievement. Students indicated positive and/or negative experiences in math based on
their perceptions of motivation and mindset in math. Students who reported that they had
a growth mindset and were motivated to learn math tended to also discuss positive
experiences in math. Furthermore, compared to students without LD, students with LD
reported more negative motivation toward their experiences with and achievement in
math. In addition, the IA numeracy scale (Gittens, 2015) was used to assess number
sense for this study. The IA numeracy scale (Gittens, 2015) was strongly correlated with
teacher evaluations of student number sense, indicating that the IA numeracy scale
(Gittens, 2015) could be a reliable source for assessing number sense for future research.

This chapter discusses these findings to provide new perspectives to deepen
understandings of the variables related to math achievement for middle school students
with LD. The discussion of findings examines a) relationships among predictors; b)
differences in students with LD and without LD on each construct; c) the relationship
between the IA Numeracy Scale (Gittens, 2015) and teacher ratings of number sense; and
d) a framework to guide deeper understandings of factors that come to bear on middle
school students’ math achievement. The examination of the findings also includes
limitations of the study and future implications for practice, policy, teacher education,
and future research.

**Discussion of Findings**

**Relationships among Predictors**
Previous affective neuroscience literature has shown that emotion and cognition are not separate constructs (LaBar & Cabeza, 2006; Ochsner & Gross, 2008; Steinberg, 2005). Previous findings suggest that motivation and mindset are noncognitive factors that are critical in academics (Blackwell et al., 2007; Dweck, 2006; Elliot & Dweck, 2005; Wigfield & Cambria, 2010). Growth mindset plays a key role in achievement by helping students be more receptive to learning and results in positive academic outcomes (Dweck, 2006; Elliot & Dweck, 2005). Motivation also has a considerable impact on academic achievement (Wigfield & Cambria, 2010). Thus, it is important to understand the complex interactions of these variables to math achievement, especially in students with LD who are at high risk to struggle in math, have low motivation and more fixed mindsets (Baird et al., 2009; Landerl et al., 2003). In this study, it was hypothesized that a lack of number sense, a cognitive construct, may interact with motivation and mindset to have a significant effect on math achievement in students with LD. This study found no statistically significant relationships among the predictors. These findings support prior evidence that number sense is related to math achievement, but do not support a significant effect of motivation and/or mindset on math achievement. This result might be due to a lack of power in the study. A small sample size was one of the primary factors that could have affected the results of this study. The magnitude of the effect of the variables may have been related to a lack of adequate power.

**Number sense and math achievement.** This study found a statistically significant correlation between number sense and math achievement ($r = .476$), indicating a large effect of number sense on math achievement. This result aligns with prior studies that number sense is one of the predictors of mathematical achievement (Akkaya, 2016;
Previous research has found that core knowledge of numbers and number relations lead to number operations and more complex numerical knowledge (National Research Council, 2009). Students with good number sense develop a quantitative intuition that helps them to solve problems in a flexible manner (Sood & Jitendra, 2013). This study found that students with low number sense scored lower in math achievement. Correspondingly, students with high number sense scored higher in math achievement, demonstrating that students who portray “critical thinking applied to a quantitative context” (Gittens, 2015, pg. 2), tend to perform higher in mathematics. On the other hand, students who lack strong number sense often need additional support to be successful in math. The present findings support past research regarding the relationship between number sense and math achievement, and highlight the need for number sense interventions for middle school students with LD.

**Implications.** Considering the central role that number sense and mathematical ability have in daily practice and the significant relationship between number sense and mathematical achievement, a thorough assessment of student’s number sense and determination of areas of need for intervention is necessary (Akkaya, 2016; Jordan et al., 2006). This study implies that this is true even into middle school, an educational context in which number sense is often ignored. For example, systematic assessment of number sense performance may help teachers determine areas in mathematics concept attainment that would benefit from additional educational support. The components of number sense in secondary school students can be targeted to improve performance in math.
Furthermore, future research can describe the role of number sense in long-term mathematical development.

**Number sense and motivation.** The statistical findings of the present study indicated that number sense and motivation had a nonsignificant correlation ($r = .223$), reflecting a small to medium effect size. However, based on their stories, some students with LD shared their frustration with math when they do not understand the content, and the boredom they experience during math instruction, which implies their low motivation to exert effort on learning tasks. When reflecting on their past experiences with math, students reported that math can be hard for them until they understand it with practice. They reported that they “enjoy” it after they understand it, and one student shared, “I wanted to do more math!” when he earned good grades in math. These statements supported the quantitative data that number sense *can* relate to how much students are motivated to learn math. The emergent themes of “enjoy math” and “desire to do more math” revealed that students tend to desire to do more math when they are successful, and unsuccessful experiences can lead to lack of inclination to practice and acquire skills (Wang & Barrett, 2013). For example, one student shared her experience, “When I was in class, and we did decimals and moved the decimal and put a period behind the whole number, that’s when I felt intelligent because I knew how to do it and I was happy. I wanted to do more.” Another student expressed, “As long as I know what to do, I feel confident.” These findings suggest that number sense may be related to students’ drive to learn more math and engage in math instruction. When students have a strong foundation of number sense, they understand numbers in more depth, which may contribute to their motivation, “students’ process of persisting to solve the problem, to attain the goal” (Deci
Ryan, 2000, p.69). Thus, the relationship between motivation and number sense for middle school students with LD deserves more attention, especially since they often have deficiencies in number sense which may be related to decreased math-related motivation. Especially for students who often face academic barriers, motivation is an important factor (Urdan & Turner, 2005).

**Implications.** Motivational beliefs impact students’ efforts on learning tasks (Wigfield & Eccles, 2000). In mathematics, when students are intrinsically motivated to learn, they tend to be more persistent, and are confident in using more challenging strategies to solve mathematical problems (Lepper & Henderlong, 2000). This present study suggests the need for future practitioners to consider improving number sense in students, especially for students who portray low motivation in math. Number sense is strongly correlated to math achievement, which implies that student’s number sense can be indirectly related to their math grades. Previous research findings suggest that achievement motivation and attitudes towards math were both directly and indirectly related to math grades (Singh et al., 2002). The results of this study suggest for future research on examining the relationship between number sense and motivation for secondary students, especially with students with LD. As the math achievement gap widens in high school (Gebhardta et al., 2014), the relationship between number sense and math achievement may become be more pronounced. As the secondary school students with LD with low number sense encounter more rigorous demands, they may also lose motivation to learn math.

**Motivation and math achievement.** The integrated analysis of the present study found a significant correlation between motivation and math achievement. Math
achievement and motivation had a nonsignificant statistical correlation ($r = .197$), reflecting a small to medium size. However, the synthesis of qualitative and quantitative data suggested a correlation between motivation and math achievement. When reflecting on their past experiences with math, students with low math achievement reported lower motivation, compared to students with high math achievement. For example, one student said, “I enjoy it a lot because it is easy and I know it most of it.” Another student stated, “I enjoy it from day to day basis. I would do it everyday basis and I am dedicated. Math is easy to me. I want to learn new things every day.” Another reported high motivation, “I am very excited because math is my favorite subject because it is the easiest to me. Sometimes I struggle but not a lot.” The findings suggest that motivation is a possible predictor of how a student can perform academically (Hodis et al., 2011). Math achievement can be a significant contributor to the developmental decline in intrinsic math motivation (Gottfried et al., 2007). The findings of the present study indicated that motivation is highly related to math achievement, which demonstrates that motivation is one of the critically important factors for academic learning and achievement across childhood through adolescence (Elliot & Dweck, 2005).

**Implications.** A significant association was found between math achievement and student motivation for mathematics. Previous evidence found that from childhood through adolescence, those with higher academic intrinsic motivation have been found to be more competent in school, generally evidencing significantly greater academic achievement, more positive perceptions of their academic competency, and lower academic anxiety (Gottfried et al., 2005). The findings of this study suggest that students who experience recurrent academic failures in mathematics can feel increasingly less
motivated to attempt their work. Repeated cycles of failure can then reinforce the idea that regardless of effort, the student will experience failure. Especially for students with LD who experience challenges and failure at a high rate (NECS, 2015), they will be more likely to have low motivation to participate in and persist through academic challenges (Morgan et al., 2008). This present study suggests the need for future practitioners to consider student’s motivation to support their math achievement. As motivation is correlated to math achievement, future practitioners can provide positive learning environments to promote motivation. In addition, the results of this study suggest that future research examine the relationship between motivation and math achievement longitudinally. The relationship between motivation and math achievement can be investigated from elementary, middle school to high school. These findings can be an important body of knowledge informing current understandings of emotional and motivational constructs and math achievement, and incorporate diverse perspectives and research foci.

Homogeneity of mindsets. Students can hold a variety of beliefs about their own intellectual abilities. Some believe that intellectual abilities are fixed, meaning they believe that abilities cannot be improved much with practice or effort, while others believe they can improve their abilities over time, with practice and efforts (Dweck, 1999). In the present study, all students reported a growth mindset toward math. The emergent themes of “intelligence comes from experience” and “you change how smart you are with practice and effort” indicate that students reported strong growth mindset orientations. One student expressed, “I was thinking that you can change how smart you are because you are learning from someone and you are able to pass that intelligence on.”
Previous research has shown that students’ own views on intelligence, fixed or growth, influence how they respond to academic challenges (Blackwell et al., 2007). Studies have also provided convincing evidence that mindset and achievement are significantly related. This study found that regardless of low or high achievement in math, all students reported that they have growth mindsets. In addition, students with LD have been found to tend toward more fixed mindsets compared to their typically developing peers (Hartmann, 2013). However, the students with LD in the present study reported growth mindsets. They believed that “with practice, they can become more intelligent.” One student with LD responded, “You can change how smart you are. I feel like I can do a lot with math problems and be successful and I have struggles. You can have struggles but you can try.” Even though many of these students had a history of academic challenges, frequent difficulty, and failure – all of which can contribute to a propensity toward more fixed mindsets – the participants with LD in this study generally reported growth mindsets.

**Implications.** The finding that every student in the present study reported a growth mindset is important for several reasons. First, it is possible that the students have similar mindsets because they attend the same school, with the same teachers, who provide similar growth mindset cultures in classrooms. Students with LD at this particular school, who scored lower in math achievement than students without LD, reported growth mindset, which is contrary to previous findings that students with LD often tend to have a more fixed mindset compared to their typical peers (Hartmann, 2013). This finding indicates how important school culture is to students’ mindsets. The school integrates a looping structure, meaning the students have the same teachers for
three years. In the three consecutive years, they establish relationships with the teachers and make deep connections. The teachers also have opportunities to know the students’ learning styles and personality types, which helps assist them to meet their students’ needs. In addition, the school has high retention rate, in which the number of students moving to another school is very low. This can also amplify the influence of school cultures on the student’s learning as they become acclimated to the culture and adopt the values of the school, such as the growth of performance shown on the data wall in classrooms, the bulletin board with growth mindset quotes, and the importance of accountability of learning. From the interview, some students attributed their intelligence to interactions with others. Some mentioned the “teacher” or “help” that they received to become smart. In addition, the culture of the accountability of learning and the importance of trying their best emphasizes the imperative role of school culture in students’ mindsets. Additionally, the small sample size in this study may not accurately reflect the tendency of students with LD toward growth or fixed mindsets. Students may have different beliefs about their own intellectual abilities that were not captured in this sample.

**Differences in Students with LD and without LD on Each Construct**

This study investigated the effects of number sense, motivation, and mindset on math achievement for middle school students with and without LD. The present study provides the first examination of this group of variables for middle school students with disabilities.

**Number sense and math achievement.** Prior research suggests that there are significant gaps in math achievement between middle school students with LD and those
without (Gebhardt et al., 2014; Mazzocco et al., 2008). This study provides additional support for those findings as the students with LD scored significantly lower than the students without LD on measures of math achievement and number sense ($t(46) = 4.653, p < 0.001; t(46) = 3.328, p = 0.002$). This present study’s findings align with previous literature that children with math learning disabilities (MLD) also have demonstrated weaker rational number knowledge than children whose difficulty with rational numbers occurs in the absence of MLD (Mazzocco et al., 2008). Considering the importance of number sense to math achievement, this gap is troubling especially for students with LD, a group who has significant challenges in both math achievement and number sense.

**Implications.** These findings suggest the need to conceptualize number sense as a possible predictor in math achievement for middle school students with LD, and thus, a potential target for intervention. It is possible that interventions aiming to improve number sense for middle school students with LD could decrease the performance gap between students with and without disabilities in secondary school. The support of number sense for students with LD can interrupt the cycle of failure and possibly help to close the gap (Blackorby, Chorost, Garza, & Guzman, 2003).

**Mindset and motivation in math.** Motivation has been described as an individual’s beliefs in their ability to carry out a specific task, goals of the individual in doing the task, and the emotional response for carrying out the task (Amrai et al., 2011). Social learning and expectancy theory research have demonstrated that students tend to actively approach activities when they feel confident and believe that they can succeed, whereas some students avoid activities when they lack confidence (Kyriacou & Goulding, 2005). Previous research also suggests that multiple psychological variables are related to
learning, such as motivation and self-efficacy (Dweck, 1999; Eccles & Wigfield, 2002), and Dweck’s research in particular, supports the notion that students are more likely to persevere through academic challenges and succeed if they have a growth as opposed to a fixed mindset (Blackwell et al., 2007; Dweck, 1999; Farrington et al., 2012). In this study, students’ perceptions of motivation indicated a relationship to their academics, and thus aligned with previous studies. Compared to students without LD, students with LD reported more negative motivation toward their experiences with and achievement in math. However, students with LD did not differ significantly from students without LD on their reported mindset.

Emergent themes overlapped between students with LD and students without LD. For the motivation scale, both group of students (with LD and without LD) overall seemed to “enjoy” math because they are learning new things and it is easy for them and reported that math can be “hard until they understand it with practice” or after a teacher “explain with examples.” The theme of “learning new things about math” provides an example of students’ experience of their excitement about learning math and their belief that it can be easier with practice and effort. However, some students with LD reported, “if I don’t get, I get frustrated and bored” on the other hand, some students without LD reported “I feel confident.” These themes for each group reflect that students with LD are less motivated to learn math when they are unsuccessful in their math classes. This is also supported by the statistical data, $F(1,46) = 4.170, p = 0.047$. They need explicit support to have positive experiences in math and be motivated to learn math.

Emergent themes revealed that there are similar levels of mindsets in relation to math achievement across both groups of students. These data revealed that both groups
(students with and without LD) interpreted intelligence as “the ability to understand quickly.” There were overlapped themes of intelligence that “intelligence comes from experience”. They believed that “intelligence comes from practice, effort, and staying attentive in class.” These emergent themes revealed that they all responded that they have growth mindsets because they believe that they can get better grades and get smarter when they try. This finding did not concur with past research that a growth mindset of intelligence can impact academic achievement (McCuchen et al., 2016; Yeager et al., 2016). Previous studies have found that fixed mindsets predominantly occurred among students low achieving students (Dweck, 2006). However, there were no significant differences in mindset between students with LD and students without LD in the present study, which does not align with prior findings.

**Implications.** This finding indicates a need for deeper understandings of the influences of motivation and mindset for middle school students, including the influence of the school’s culture. The student’s shared experiences describe how a school’s culture might affect student’s motivation and mindset. In addition, the integrated analyses provide new insights into how teachers and administration should approach intervening on mindset and motivation in a school that already values that as part of their culture. School personnel need to understand the importance of school culture on student’s mindsets and motivation, and establish positive learning environments to cultivate growth mindset and motivate students. For example, schools can develop value systems that focus on positive student interactions by promoting growth mindsets and motivation, especially for students with LD. Future studies can investigate these complex
relationships among these variables to better understand how school cultures can interact with mindset and motivation to support academic achievement.

**IA Numeracy and Teacher Ratings**

Despite a strong U.S. emphasis on math achievement, there are very few studies focusing on a foundational component of math achievement for middle school students: number sense. Prior studies of number sense focus primarily on younger groups of students, primarily preschool and elementary school students (Aunio et al., 2006; Irvendi, 2011; Locuniak & Jordan, 2008; Schneider et al., 2008; Xu et al., 2005). The late elementary and middle school student population is underrepresented in the number sense literature. This critical gap is at least partly due to a lack of valid and developmentally appropriate instruments for upper-elementary and middle school students (Gittens, 2015). In an attempt to fill the gap in the literature regarding number sense in students in upper elementary and middle school, this study used a new scale for assessing numeracy as an applied form of critical thinking among children and adolescents, the IA Numeracy Scale (Gittens, 2015). In Gitten’s study, number sense was defined as numeracy as “critical thinking applied to a quantitative context” (Gittens, 2015, pg. 2), and operationally defined it as the ability to critically reason about quantitative information.

Based on this definition, teachers at the school sampled in the present study rated students’ number sense, and their ratings were strongly correlated with scores on the IA numeracy scale (Gittens, 2015; ICC= .885, a 95% CI .814 to .932, F(47,94) = 8.692, \( p < 0.001 \)). In addition, the teacher evaluations from three teachers were also strongly correlated, indicating good interrater reliability. This finding, while preliminary, may support the reliability of the IA scale and also teacher evaluations in assessing number
sense of middle school students with LD. The correlation may ensure validity in the assessment. The high interrater reliability suggests potential validity of this assessment tool for future use. This validation is important in future investigation of number sense for middle school students.

**Revised Conceptual Framework**

Prior literature has suggested that cognitive, emotional, and motivational variables can be critical contributors to math achievement (Akkaya, 2016; Elliot & Dweck, 2005; Ivrendi, 2011; Xu et al., 2005). This study aimed to explore how these cognitive, emotional, and motivational constructs are related to math achievement for students with LD. In the original framework for this study (see Figure 1), the bi-directional arrows suggested that the variables are related to math achievement. A review of the literature suggested theses as potential correlates of math achievement for middle school students with LD. However, the findings of the present study revealed that those relationships may be different.

In the revised framework based on the present findings, number sense is statistically significantly correlated to math achievement, and through the integrated analysis, there were differences in motivation between the two groups (students with LD and students without LD) and motivation was correlated to number sense and math achievement. The solid lines indicate significant relationships that were found in the study. The dotted lines depict the insignificant relationships among the variables. The question marks suggest future implication to be investigated (see Figure 4).
Figure 1. The hypothesized relationships between mindset, number sense, and motivation which are thought to be different based on disability status to math achievement.

Figure 4. The relationships between mindset, number sense, and motivation which are thought to be different based on disability status to math achievement, based on the findings of this study.
Limitations

As with all research with small sample sizes, it is difficult to validate the findings in the field of special education. Hence, I would caution the readers in applying the findings broadly to the field of special education. This study has several limitations.

The participants were recruited from the same school, a school which has a distinct culture and shared similar experiences. Many of the participants have assimilated the school values and climate of positive academic attitudes, and reflect the school values of high motivation and growth mindset. Most of the students have attended this school for at least several years, and many of the middle school participants have been in the same school since kindergarten and share similar academic cultures. For this reason, having LD might not be a significant risk factor for low motivation and fixed mindset, as it has been in past studies. Collecting data from only one school was a limitation to this study. In the future, working with numerous schools and collecting data from diverse school cultures will result in more generalizable findings.

The participants shared their experiences in their math classes. They explained their perceptions related to their positive and/or negative experiences in math. Considering the vulnerability of sharing their personal experiences, it is important to consider that the students only shared what they were comfortable sharing. In addition, the interviewer was one of their teachers. For this reason, there is the possibility that students might have shared experiences that would satisfy the interviewer. To overcome this limitation, future research should utilize a design in which the participants do not know the interviewer.
For the group of participants with LD, only their diagnoses of LD were included in their demographics. For future study, a detailed information of their demographics, such as their math/reading disabilities, could be meaningful in drawing more valuable conclusions about the population by the specificity of their disabilities, and suggest the potential significance of math disabilities on number sense, mindset, and motivation in relation to math achievement.

The measurement tool for number sense is a newly created assessment that needs further validation. There is a lack of valid and developmentally appropriate instruments to measure number sense for upper-elementary and middle school students. At the current time, the IA scale has shown to be reliable; however, the field is in need of more validated instruments to measure number sense accurately.

Finally, this study was underpowered. This lack of power adds another limitation to this study. The small sample size utilized in this study meant that only very large effects of each variable would be able to be detected. It is possible that small to medium effects among the variables went undetected in this study due to low sample size and low power.

**Recommendations**

**Recommendations for Practice**

The results of this study may provide guidance for recommendations for practice in the field of special education. One implication of this study is that practitioners should consider novel interventions and classroom practices for middle students with LD to support math achievement. The findings from this study demonstrate the students with LD score lower than students without LD on standardized math tests. This result aligns
with prior literature that there are significant gaps in math achievement between middle
school students with LD and those without (Gebhardt et al., 2014; Mazzocco et al.,
2008). Mathematical difficulties are widespread in school-age children and adolescents,
and up to 20% of individuals have some form of mathematical learning disability (MLD)
(Butterworth et al., 2011; Dowker, 2009; Parsons & Bynner, 2005). Individuals with
MLD have specific difficulties with numerical and arithmetic problem solving, despite
age-appropriate schooling and absence of impairments in other cognitive domains
(Butterworth et al., 2011). While there is an emphasis on math achievement as an
important target in the field of education, it is critical to find a way to carefully scrutinize
factors related to mathematics achievement, including number sense. This scrutiny has
the potential to lead to effective intervention planning appropriate for students with LD.

For students with LD, effective math instructional techniques and
accommodations can be incorporated in the classrooms. Under Section 300.160 of IDEA,
for participation in assessments, each State must ensure all students with disabilities are
included in all general State and district-wide assessment programs, including
assessments with appropriate accommodations as indicated in their respective
individualized education programs (IDEA, 2004). IDEA and its regulations do not define
accommodations, but it is generally agreed upon that, accommodations allow a student to
complete the same assignment with a change in timing, formatting, setting, scheduling, or
presentation. Students with LD are often provided with such accommodations to support
them in their classrooms, such as the use of a calculator during assessment. However,
when applying the conceptual understanding in math, the students need foundational
skills to apply in solving problems and use the calculators appropriately. Thus, the
instructions need to focus on cultivating the foundational skills and teaching students to apply those skills with an aid of calculator.

Furthermore, in secondary schools, many schools have block schedules, in which students may not have math classes on a daily basis. This environment can have an impact on the students, where they have limited instructional time in math classes. Thus, practitioners should consider using numerous resources to assist in developing instructional accommodations and finding appropriate materials to support the students by closing the gap in math, such as using the resource time on a daily basis for their block schedule to support their foundational skills in small groups, cultivate their positive motivational skills, and promote their growth mindset, which in turn, can promote success in their math achievement.

Practitioners should also consider various factors that have been found to be critical in math achievement and focus on the factors to support the students effectively. To meet the global goal of workforce readiness, competitiveness, and equity in access, the United States has been devoting effort to accelerate mathematics instruction (National Research Council, 2011). In addition to standard mathematics instruction, there are other variables that can be addressed through targeted intervention to enhance achievement. For example, recent research suggests that cognitive, emotional, and motivational variables can also be critical contributors to math achievement (Cantlon et al., 2009; Elliot & Dweck, 2005; Gottfried et al., 2005; Sasanguie et al., 2013). It is critical to consider cognitive, emotional, and motivational constructs related to math achievement in classroom practices to support students with learning disabilities. For example, some of the students in the interview mentioned the “teacher” and “help” that they receive at
school. They stated that math can be easier with more help and also the support that they receive from the teachers can boost their confidence and motivation to learn.

**Recommendations for Policy**

This study has several implications for policy recommendations regarding valid screening in math achievement for middle school students with LD. Policies should encourage school personnel not only to focus on academic components related to math achievement, but also focus on cognitive and noncognitive components to help identify students in need of support and provide appropriate interventions for them. To fill the gap in math achievement for middle students with LD, policies must emphasize the holistic assessment of students, which includes cognitive (number sense) and also noncognitive factors (motivation and mindset) that can be easily overlooked in education. Thus, this study highlights the need for screening processes and also interventions to address constructs that can affect math achievement.

In addition to policy makers, intervention providers, educators, and school psychologists need to work altogether to create appropriate screening processes and targeted protocols to effectively assess and support middle school students with LD in math achievement before they enter high school. Middle school is a developmental period during which students engage in increasingly complex cognitive and emotional tasks (Balfanz, 2009). It is a period in an adolescent’s life during which they are moving toward a higher level of cognition, but some students enter this period with gaps in their foundational knowledge. As the gap widens, students may face challenges in the cognitive demands of mathematics learning. These challenges can impact adolescent identity, competence, and motivation (Wigfield & Wagner, 2005). Effectively supporting
students may include developing strategies to improve math achievement by way of “filling the gap” in number sense to cultivate foundational mathematical knowledge.

**Recommendations for Teacher Education**

These findings also indicate that pre-service teachers should be taught the importance of number sense for middle school students with LD. Teacher preparation programs should recognize that there is a gap in math achievement for middle school students with LD due to weak number sense. They need to understand the rigor of the Common Core standards that the students are held to and the widening gap that overwhelms both teachers and students. This can allow teacher preparation programs to focus on these foundational skills and explore number sense in depth to teach pre-service teachers how to design effective instructions for middle school students with LD. Thus, the pre-service teachers will be trained to support the students who meet these challenges.

**Recommendations for Future Research**

In order to address some of the limitations of the present study, several recommendations are suggested for future research. There are numerous studies on number sense in elementary school, because that is the period when students build math foundations. However, there is a critical gap in the literature regarding number sense in middle school students LD. Future research should investigate the validity of the IA scale as a measure of number sense. Moreover, future research can investigate number sense for the middle school population to create a valid, evidence-based assessment that truly reflects number sense at this developmental time.

Another area of recommendation involves collecting data from various schools to study diverse samples. Diversity in the sample would improve the generalizability of the
findings. In addition to studying a wide range of schools, researchers should also investigate cognitive, motivational, and emotional constructs in relation to math achievement longitudinally. This might elucidate the development of number sense over time for students with LD. Future research could also investigate the effect of number sense on graduation rates by tracking students over the long-term. Likewise, future studies can investigate whether motivation and mindset are statistically significantly related to math achievement for middle school students with LD and without LD. This longitudinal work could offer important insights for the field of special education and inform policymakers, educators, future researchers about the crucial importance of these factors related to math achievement.

**Conclusion**

Mathematical difficulties are widespread in school-age children and adolescents (Butterworth et al., 2011). Despite a strong U.S. emphasis on math achievement, math scores in the U.S. are falling (NECS, 2015). While there is an emphasis on math achievement as an important target in the field of education, we need to find a way to carefully scrutinize factors related to mathematics achievement. We need to explore and determine strategies to improve it, especially for middle school students with learning disabilities.

The purpose of this study was to explore relationships between math achievement and three variables: mindset, motivation, and number sense for middle school students with and without LD. The focus was to explore how these variables contribute to variance of math achievement among middle school students with and without LD. It aimed to fill a gap in the math achievement literature by providing the first examination
of this group of variables concurrently for middle school students with LD. Though the results of this study are limited by the small group of participants, it reveals the need to investigate the constructs of interest in future work. This study offers potential expansions of understanding of factors that influence math achievement in the middle schoolers. Before students enter high school and learn more complex concepts, they need a strong foundation, cognitively, emotionally, and motivationally. Middle school is a critical transitional period between elementary to high school, in which there is an opportunity to address foundational constructs, prior to the more rigorous demands of high school. We need to give more attention the middle school students to help them progress on their path to success.
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APPENDIX: MEASURES

MATH MOTIVATION SCALE

Mastery goal structure

1 (not at all true for me) to 5 (very true for me)

Learning a lot of new things is what is important to me in math.

One of my main goals in math is to improve my skills.

My main goal in math is to learn as much as I can.

Really understanding my math work is important to me.

Learning new skills in math is one of my goals.

Academic self-efficacy for math

1 (not at all sure) to 5 (very sure)

How certain are you that you can learn everything taught in math?

How sure are you that you can do even the most difficult homework problems in math?

How confident are you that you can do all the work in math class, if you don’t give up?

How confident are you that you can do even the hardest work in your math class?
APPENDIX: MEASURES

MINDSET SCALE

This questionnaire has been designed to investigate ideas about intelligence. There are no right or wrong answers. We are interested in your ideas. Using the scale below, please indicate the extent to which you agree or disagree with each of the following statements by writing the number that corresponds to your opinion in the space next to each statement.

1 = strongly agree
2 = agree
3 = mostly agree
4 = mostly disagree
5 = disagree
6 = strongly disagree

_____ You have a certain amount of intelligence, and you can’t really do much to change it.

_____ Your intelligence is something about you that you can’t change very much.

_____ No matter who you are, you can significantly change your intelligence level.

_____ To be honest, you can’t really change how intelligent you are.

_____ You can always substantially change how intelligent you are.

_____ You can learn new things, but you can’t really change your basic intelligence.

_____ No matter how much intelligence you have, you can always change it quite a bit.

_____ You can change even your basic intelligence level considerably.
APPENDIX: MEASURES

INTERVIEW PROTOCOL

First Interview: Motivation

I’d like to ask you about how motivated you are about learning math. Everyone has different thoughts and I want to know what you think. There is no right or wrong answer. I just want to know what you think about math.

1. How much do you enjoy doing math?
2. When you walk into a math class, how confident are you that you are going to get a good grade?
3. How excited are you to learn math?
4. Tell me about what is easy about math
5. Tell me about what is hard about math.
6. What were you thinking when you were answering the question, “How sure are you that you can do even the most difficult homework problems in math?” on the survey?

Second Interview: Mindset

I’d like to also ask some questions about being smart or intelligent. Remember, everyone has different ideas and I want to know what you think.

1. What does it mean to say someone is smart?
2. Where do you think someone’s intelligence comes from?
   Probes:
   • Do you think he or she is born intelligent?
• Is it from his or her experience?

3. Tell me a story about when you were feeling intelligent.

Probes:

• What happened?
• What did you do?
• How did you feel?
• What did you want to do when you felt intelligent?

4. What were you thinking about when you were answering, “To be honest, you can’t really change how intelligent you are” in the survey?

5. What are some recent situations in your life including school that make you feel more that you have a growth mindset or a fixed mindset (define terms if needed)?

   In general, do you think you have more of a growth or fixed mindset in life?
   Explain.

Thank you for your time and sharing your thoughts with me.
APPENDIX: MEASURES

TEACHER EVALUATION

How strong is the student’s ability to apply critical thinking to a quantitative context?

1 = low
2 = moderately low
3 = average
4 = moderately high
5 = high
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Table 1. *A table summarizing different number sense assessments for different age groups*
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</tr>
<tr>
<td>Sasanguie, D., Gobel, S. M., Moll, K., Smets, K., &amp; Reynvoet, B. (2013)</td>
<td>71</td>
<td>6-8 year</td>
<td>42</td>
<td>29</td>
</tr>
</tbody>
</table>
Xu, F., Spelke, E. S., & 16 5-6 months


Table 2. Selected Characteristics of the Study Participants
<table>
<thead>
<tr>
<th>Definition of Number Sense</th>
<th>Number Sense Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akkaya, R. (2016)</td>
<td>Number sense is a complex process involving certain associations and skills related to numbers: assess skills demonstrated by an individual during this process</td>
</tr>
<tr>
<td>● an understanding of different relationships between numbers and operations and the flexible use of these relationships</td>
<td></td>
</tr>
<tr>
<td>● divided into sub-components:</td>
<td></td>
</tr>
<tr>
<td>a. knowledge of numbers</td>
<td></td>
</tr>
<tr>
<td>b. use of multiple representations of numbers</td>
<td></td>
</tr>
<tr>
<td>c. ability to grasp number magnitudes</td>
<td></td>
</tr>
<tr>
<td>● development of number sense is a combination of progress in general and specific skills</td>
<td>● specific numerical skills build on general numerical skills and are mostly affected by social, environmental and cultural factors</td>
</tr>
<tr>
<td>● central conceptual structure for whole numbers, a cognitive structure that permits a child to interpret the world of quantity and numbers to acquire new knowledge in this domain, and to solve the range of problems that it presents</td>
<td>● the development of general skills, in contrast, is more dependent on maturation than on direct influence</td>
</tr>
<tr>
<td></td>
<td>● general skills may also be influenced by factors other than maturation</td>
</tr>
</tbody>
</table>

depth understanding the meaning of numbers

- a nonverbal system of numerical representation, originating in infancy, continues to function throughout childhood and into adulthood for the rapid assessment of nonsymbolic numerical quantities
- there is a unitary system for representing and/or comparing symbolic and nonsymbolic numerical values
- there is a common network of frontal and parietal regions that subserves numerical operations in older children and adults

Ivrendi, A. (2011)

- an essential factor in fluent utilization of mathematical knowledge, fostering the capability to work out arithmetic calculations, solving applied mathematics problems, and progressing from simple to more complex number knowledge when acquiring formal mathematics knowledge
- fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and look at the world and make counting, number knowledge, number transformation, estimation, and number patterns are used
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Key Points</th>
</tr>
</thead>
</table>
| Jordan, N. C., Kaplan, D., Nabors Olah, L., & Locuniak, M. N. (2006) | | ● understanding of whole numbers, number operations, and number relations  
● recognition that numbers represent quantities and have magnitudes, that counting is guided by principles related to one-to-one correspondence, fixed order, and cardinality, and that sets can be transformed through addition and subtraction |
● ability to subitize (ability to 'see' a small amount of objects and know how many there are without counting) small quantities, to compare numerical magnitudes, to count, and to perform simple arithmetic calculations |
| Sasanguie, D., Gobel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013) | | approximate number sense is the basis of the arithmetic skill, to compare, add, and subtract different dot arrays |
| Schneider, M. et al. (2008). | | ability to quickly understand, approximate, and manipulate numerical quantities |

Number competencies are affected by counting, number knowledge, and number operations. Development of fluency on number combinations seems to be constrained by weakness in accessing, comparing, and mentally manipulating number representations. Symbols are thought to acquire meaning by being associated with the preexisting nonsymbolic approximate representation. The “mental number line” is regarded as the core neurocognitive system underlying number sense: it underlies a variety of behavioral competencies, like...
estimating, computing, and efficiently using notational systems to solve mathematical problems

<table>
<thead>
<tr>
<th>Toll, S. M., Kroesbergen, E. H., &amp; Van Luit, J. H. (2016)</th>
<th>an intuitive understanding of numbers, their magnitude, relationships, and how they are affected by operations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-symbolic number sense, or the understanding and ability to discriminate quantities, is thought to be based on a cognitive system dedicated to processing quantity information</td>
</tr>
<tr>
<td></td>
<td>symbolic number sense is linking such non-verbal representations to the corresponding verbal representations</td>
</tr>
<tr>
<td></td>
<td>manipulating numbers and the quantities they represent</td>
</tr>
<tr>
<td>Xu, F., Spelke, E. S., &amp; Goddard, S. (2005)</td>
<td>a sense of approximate numerical magnitudes</td>
</tr>
<tr>
<td></td>
<td>discriminability between two numbers that depends on their ratios</td>
</tr>
<tr>
<td></td>
<td>explore the ratio limit in infants with numerosities</td>
</tr>
<tr>
<td></td>
<td>infants are sensitive to numerosity only when the sets they encounter are large</td>
</tr>
</tbody>
</table>

Table 3. Definition of Number Sense
<table>
<thead>
<tr>
<th>Study</th>
<th>Research Question</th>
<th>Measure</th>
<th>Research Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akkaya, R. (2016)</td>
<td>To examine number sense performance of secondary school students in terms of the</td>
<td>descriptive survey design was to determine the number sense performance of secondary school students who had received no prior education on number sense</td>
<td>The “Number Sense Test” (NST) was used to assess the number sense performance of secondary school students: comprised of 50 multiple choice questions, with questions for each of the 5 sub-components of number sense (number concept, multiple representations, understanding and using the equivalent expressions of numbers and figures, understanding the effect of operations on numbers, and flexibility in making calculations)</td>
</tr>
<tr>
<td>Aunio, P., Niemivirta, M., Hautamaki, J., Van Luit, J. H., Shi, J., &amp; Zhang, M. (2006).</td>
<td>To examine the influence of nationality and age on Chinese and Finnish children’s number sense was assessed using the Utrecht Early Numeracy Test</td>
<td>The Utrecht Early Numeracy Test taps several aspects of young children’s numerical and non-numerical knowledge of quantity. It includes eight separate scales for assessing:</td>
<td></td>
</tr>
</tbody>
</table>
concepts of comparison, classification, seriation, the use of number words, structured counting, resultative counting, one-to-one correspondence, and general understanding of numbers.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Study Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantlon, J. F., Libertus, M. E., Pinel, P., Dehaene, S., Brannon, E. M., &amp; Pelphrey, K. A.</td>
<td>2009</td>
<td>To examine the brain mechanisms that 6- and 7-year-old children and adults recruit to solve numerical comparisons across different notation systems, subjects were presented with two numerical stimuli simultaneously on a computer monitor and pressed a button corresponding to the side of the screen hosting the larger numerical value conducted fMRI scans as subjects performed a number comparison task to measure the brain response to numerical judgments across notation systems in children and adults.</td>
</tr>
<tr>
<td>Ivrendi, A.</td>
<td>2011</td>
<td>To examine predictive power of behavioral self-regulation, family and child characteristics on children’s number sense, two curriculum specialists examined the Assessing Number Sense Instrument (ANS) for the content validity of the items, ANS consisted of items on number production, number identification, and counting, HTKS measures three dimensions of behavioral self-regulation: inhibitory control, attention, and working memory, children’s behavioral self-regulation was measured with an instrument entitled, Head, Toes, Knees and Shoulders (HTKS).</td>
</tr>
<tr>
<td>Jordan, N. C., Kaplan, D., Nabors Olah, L., &amp; Locuniak, M. N.</td>
<td>2006</td>
<td>developed and tested a purposeful eight-week number sense intervention for whole number concepts related to a pretest, post test, and delayed post test design.</td>
</tr>
</tbody>
</table>
Kindergartners from low-income families were targeted for instruction that included consistent representations (primarily chips, black dots, and fingers) and centered activities on a number list from one to ten. Finger counting was an area of particular focus. Fingers are an accessible resource for representing numbers and counting. The Number Sense Brief (NSB) is an untimed measure, which assesses counting skills and principles, number recognition, number knowledge, and number operations. General mathematics achievement was assessed using the Woodcock-Johnson III Tests of Achievement.

Locuniak, M. N., & Jordan, N. C. (2008) investigated number sense in kindergarten as well as skill on related tasks assessing reading, oral language, memory, and spatial skill. They used two memory measures to compare relative contributions of simple short-term memory versus working memory.

The number sense battery included varied number sense tasks (counting, number knowledge, nonverbal calculation, story problems, and number combinations). Reading was assessed with the sixth edition of the Dynamic Indicators of Basic Early Literacy.
Literacy Skills

- memory span was assessed with the fourth edition of the *Wechsler Intelligence Scale for Children* (WISC-IV) Digit Span subtest
- calculation fluency was measured with the “Assessment of Math Fact Fluency” (children were presented with an addition fluency measure and comparable subtraction test)

Sasanguie, D., Gobel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). To examine three domain-specific numerical processes as possible predictors for individual differences in mathematics achievement: acuity of the ANS, performance in symbolic number comparison, and accuracy of number–space mappings of children. All children first performed the symbolic and nonsymbolic number line estimation tasks, followed by the symbolic and nonsymbolic comparison tasks. Results of their standardized tests (timed arithmetic test, general curriculum-based math test, and in a number line estimation task, participants are instructed to indicate the position of a number on an empty number line. In a nonsymbolic task, participants were asked to indicate without counting which of the arrays consisted of more...
curriculum-based spelling achievement test) were used as the dependent variable. • squares by pressing the left or right button on the keyboard for symbolic comparison task, participants needed to select the larger number: the stimulus set consisted of numbers 1 to 9, but only combinations of stimuli with a maximum distance of 5 between both numbers were presented to the children.


- addition accuracy: 20 addition trials with two-digit addends
- estimation accuracy: solve 30 trials of the number line estimation task
- fixation accuracy: children were instructed to actively search for and focus their gaze on the correct position
For each number:

a. The answer patterns from children's estimate of the positions of given numbers on an external number line where only the starting and the end points were labeled.

b. The use of eye movements was analyzed for investigating children's use of the number line.

Toll, S. M., Kroesbergen, E. H., & Van Luit, J. H. (2016). To test the hypothesis that both visual working memory and number sense affect math performance within a longitudinal time span of 2 years.

- The children were screened on visual working memory (Automated Working Memory Assessment) and number sense.
- Visual working memory and the non-symbolic number sense test were administered on the computer.
- Symbolic number sense test was a pencil-and-paper test.
- In the non-symbolic task, Dot Comparison, children were asked to compare two arrays with dots and had to indicate the array with the greatest number of dots.
- The symbolic task consisted of twenty items from the Early Numeracy Test-Revised (4 components: using numerals,
two years later, the children’s mathematical performance was examined with two different tests synchronized and shortened counting, resultative counting, and general understanding of numbers

mathematical performance was measured with the same tests as used in Kroesbergen and Van Dijk (2015) – a math facts and math problems test

Xu, F., Spelke, E. S., & Goddard, S. (2005) To investigate 6-month-old infants’ capacity to represent numerosity in visual-spatial displays

infants’ discrimination of larger numerosities under conditions that control for all the continuous quantities to which infants might be sensitive

infants’ numerosity discrimination that demonstrates the set-size ratio signature found in adults, children and non-human animals

tested 6-month-old infants’ discrimination between arrays of 16 versus 32 discs and arrays of 16 versus 24 discs

tested infants’ large number discrimination with controls for contour length, measured

tested infants’ small-number discrimination with controls for total filled area, display size and density, and element size

Table 4. Research Question and Design
<table>
<thead>
<tr>
<th>Study</th>
<th>Analysis</th>
<th>Findings</th>
<th>Strengths</th>
<th>Weaknesses</th>
<th>Important Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aunio, P., Niemivirta, M., Hautamaki, J., Van Luit, J. H., Shi, J., &amp; Zhang, M. (2006)</td>
<td>descriptive statistics for the individual items within both nationalities and across gender</td>
<td>the Chinese children obtained higher scores on both the relational tasks and the counting tasks as a function of age</td>
<td>the test items clearly reflect two different aspects of the number-sense construct: the children’s ability to organize and compare given information and their ability to use and operate with number-word sequence.</td>
<td>complex approach in analyzing the bi-dimensional structure of the patterning of differences clearly supports the validity of distinguishing the two forms of number sense.</td>
<td>the natures of relational and counting skills are somewhat different: relational skills reflect general numerical abilities, less influenced by direct teaching. Unlike relational skills, counting skills rely on the use of a culturally based symbolic system. The development of informal mathematical skills is universal, but the...</td>
</tr>
</tbody>
</table>
pace of development may vary


compared patterns of brain activity between children and adults during judgments of the symbolic and nonsymbolic numerical stimuli to identify developmental differences in the brain regions recruited to solve numerical problems.

- performance on Arabic numerals was higher than on dot arrays.
- For adults, region evoked significant response to both Arabic numerals and dot arrays, a significant numerical ratio effect included the pre-central gyrus and superior parietal cortex.
- Children exhibited ratio-dependent activity in the IFG,

- provides novel information regarding the mechanisms underlying children’s representation of “numbers” across visually distinct inputs such as arrays of dots or symbolic characters.
- can be used to support brain-education-mind interdisciplinary fields.
- demonstrates several key differences in the functions of this network between adults and children that implicate a role for the IFG in the early developmental stages of notation-independent numerical processing.

• a core neural system processes the values of
the pre-central gyrus, and the thalamus.

- Unlike adults, children did not exhibit a significant ratio effect in superior parietal cortex.

| Irvendi, A. (2011) | multiple regressions using stepwise method were computed on the data. Independent variables (self-regulation, age, gender, mothers’ education level were significant predictor variables in the final regression model). ANS scores were regressed on HTKS, age, education level of mothers and gender. The validity of the assessment tools: ANS and HTKS (only two people checked for validity) to relate to number sense. It studies the relationship between descriptive and cultural characteristics of the population. A statistically significant relationship among self-regulation of behavior, mothers’ level of education, age, and gender and children’s performance on number sense. The association |
entered in the model using the stepwise method between learning and self-regulation of behavior seems to be critical for the development of symbolic mathematical knowledge (symbolic knowledge develops through formal instruction).

Jordan, N. C., Kaplan, D., Nabors Olah, L., & Locuniak, M. N. (2006) a series on one-way ANCOVAs were conducted (intervention group vs. control) for the WJ, the results were obtained significantly higher and meaningful adjusted outcome scores at posttest as well as at delayed posttest. 

● the intervention group needs to look at the long-term effect

● captured on a number sense assessment tool that is sensitive to short-term progress

● an intervention can possibly support students with the study is focused and provides some useful information

● the structure and the organization of the study is easy to understand

● for the WJ, the results needs to look at the long-term effect

● captured on a number sense assessment tool that is sensitive to short-term progress

● an intervention can possibly support students with the study is focused and provides some useful information

● the structure and the organization of the study is easy to understand

● for the WJ, the results
were significant at posttest but not a delayed posttest and only for the calculation problems subject.

Locuniak, M. N., & Jordan, N. C. (2008) raw scores were used for all analyses.

- simple correlations among the number measures contributed a significant amount of variance.
- block entry regression analysis allowed to predict calculation fluency in second grade, taking into account general cognitive and reading fluency measures.
- varied number sense tasks allow to look at the relative importance of different but related quantitative skills to calculation fluency.
- the findings of relation of early number skills to later calculation fluency with basic operations is not sufficient for learning high-level mathematics.
- fundamental number concepts and procedures does not affect fact mastery.
- the added benefits in a relatively short time period is important.
and number sense variables

Sasanguie, D., Gobel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013) examined the relation between the indexes of the different experimental measures.

- a. for the nonsymbolic comparison task, the Weber fractions and the mean accuracies were used as indexes.
- b. for the symbolic comparison task, we used the median reaction times and the individual distance effects.
- c. as indexes for the number line estimation task and math achievement scores for the general curriculum-based math test measuring a broader spectrum of skills was found.

- observed associations between experimental tasks and math ability achievement
- number sense in different tasks broken down into topics, easy to understand
- studies and details about each experimental tasks

- emphasized the importance of learning experiences with symbols for later math abilities
- number sense can be described in different experimental tasks

- performance on both mathematics achievement tests was best predicted by how well children compared digits
- an association between performance on the symbolic number line estimation task and math achievement
tasks, the symbolic and nonsymbolic mean percentages of absolute error were used.

Schneider, M., Heine, A., Thaler, V., Torbeyns, J., De Smedt, B., Verschaffel, L., & Stern, E. (2008). Eye-movement data were collected using a stationary eye-tracking system. Addition, estimation, and fixation accuracy were assessed. Eye movements reflect children’s increasing knowledge about natural numbers, their interrelations, and ways of their spatial representation. These eye movements emphasize the relation between number line estimation and another important mathematical competence, addition, and competence, addition. Eye-movement data can reflect additional factors, that is, measurement error, in addition to actual competence. Understanding of natural numbers lies at the very core of children’s number sense, a prerequisite for their future acquisition of more advanced mathematical concepts. Eye movements reflect children’s increasing number sense.

Elementary school children’s eye-tracking data are less reliable than the data of older children and adults.

Eye-movement data can reflect additional factors, that is, measurement error, in addition to actual competence. Understanding of natural numbers lies at the very core of children’s number sense, a prerequisite for their future acquisition of more advanced mathematical concepts.

Eye movements reflect children’s increasing number sense.
allow for a direct investigation of how children orient themselves in problem situations and how they direct their attention to specific features.

| Toll, S. M., Kroesbergen, E. H., & Van Luit, J. H. (2016) | correlation and multiple regression analyses were conducted to examine the relation between visual working memory, non-symbolic number sense, symbolic number sense, and mathematical performance in first grade. | multiple regressions revealed that both visual working memory and symbolic number sense are predictors of mathematical performance in first grade. | a longitudinal study over the time span of 2 years could have included verbal working memory, because defined number sense has been found to have a unique association with non-symbolic and symbolic mathematical skills as long-term predictors of mathematical performance in early primary school. | contributes to the knowledge on visual working memory and (non-)symbolic number sense as long-term predictors of mathematical performance in first grade. |
the strongest predictor for both math areas (math facts and math problems)

- non-symbolic number sense only predicts performance in math problems
- multivariate analyses of variance showed that a combination of visual working memory and number sense deficits (NSDs) leads to the lowest performance on mathematics.
Xu, F., Spelke, E. S., & Goddard, S. (2005) mixed-factor analysis of variance (ANOVA) testing the between-subject factor

- infants successfully discriminated between arrays of 16 versus 32 discs, but not 16 versus 24 discs
- infants successfully discriminated the large-number displays but showed no evidence of discriminating the small-number displays.

Findings add empirical support to the thesis that a common system of number representation, shared by humans and other animals, is present and functional in 6-month-old infants. Changes in looking time from habituation to test are not reliable indexes of infants’ responses to changes in number. These findings provide evidence that infants’ discrimination shows the set-size ratio signature of numerosity discrimination in human adults, children, and many non-human animals. They provide evidence that infants have robust abilities to represent large numerosities.

Table 5. Analysis and Findings